

Noise-Induced Dynamics in a Multistable Erbium-Doped Fiber Laser: A Short Review From Extreme Events to Photonic Information Processing

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Abstract

Noise is often regarded as a source of instability and degraded performance in optical systems. In nonlinear dynamical systems, this view is incomplete: fluctuations can reveal critical susceptibility, activate transitions among coexisting attractors, generate rare extreme excursions, and enable probabilistic information processing. This review develops a unified narrative around diode-pumped erbium-doped fiber lasers (EDFLs), a compact photonic platform in which multistability, bifurcations, attractor hopping, prebifurcation noise amplification, rogue-wave generation, deterministic Brownian-like dynamics, symbolic diagnostics, optical logic, and data-driven stochastic cloning have all been investigated. We first establish the EDFL as a class-B nonlinear oscillator and review its reduced rate-equation description, emphasizing the roles of pump modulation, amplified spontaneous emission, and basin geometry. We then examine noise-induced hopping among periodic attractors and the amplification of fluctuations near saddle-node, period-doubling, and crisis bifurcations. These mechanisms lead naturally to extreme-event physics, where filtered noise promotes rare visits to high-amplitude subharmonic states and produces non-Gaussian rogue-wave statistics. Finally, we review how noise-controlled state occupation can be used for optical logic and how reservoir computing differentiates strong trajectory cloning from weak statistical cloning in partially observed stochastic systems. The main conclusion is that noise in a multistable EDFL is simultaneously a perturbation, a probe of hidden dynamical structure, and a controllable resource for photonic information processing.

Keywords— erbium-doped fiber laser; nonlinear dynamics; multistability; noise-induced intermittency; bifurcation; rogue waves; reservoir computing; optical logic gates

1 Introduction

Noise is commonly associated with uncertainty, limited reproducibility, and reduced signal quality. This association is justified in many practical systems, but nonlinear dynamics provides a more nuanced picture. In the presence of multiple stable attractors, noise can induce transitions between qualitatively distinct states; near a bifurcation, it can be strongly amplified; and in metastable regimes, it can make rare high-amplitude excursions observable. Consequently, the effect of noise is determined not only by its variance or amplitude but also by the geometry of the deterministic phase space, the local stability of attractors, the location of basin boundaries, the distance to critical transitions, and the temporal spectrum of the forcing [1, 2].

This broader perspective has been developed in many physical contexts. Noise-induced switching between attractors, stochastic resonance, coherence resonance, critical amplification, and noise-induced intermittency show that fluctuations can produce structured dynamical responses rather than merely featureless disturbances [3, 4, 5, 6]. The challenge is to identify the deterministic mechanism through which noise is transformed into a measurable response. A particularly effective strategy is to use a physical system that is simultaneously nonlinear, experimentally controllable, and sufficiently well modeled to connect observations with state-space dynamics.

Lasers fulfill these criteria. Their dynamics may include locking, period doubling, quasiperiodicity, chaotic oscillations, synchronization, and coexistence of multiple attractors. The experimental discovery of subharmonic bifurcations, multistability, and turbulence in a Q-switched gas laser provided an early demonstration that lasers are useful laboratories for nonlinear dynamics [7]. Fiber lasers are especially attractive because they are compact, efficient,

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compatible with fiber-optic components, and relevant to communications, sensing, reflectometry, medicine, and metrology [8, 9]. Erbium-doped fiber lasers, in particular, operate in the technologically important telecommunications region and possess a rich interplay between gain, loss, population inversion, cavity feedback, spontaneous emission, and external pump modulation.

The present review focuses on a connected body of work involving diode-pumped EDFLs subjected to harmonic pump modulation and externally added noise. The sequence begins with experimental evidence that noise can cause hopping among coexisting periodic attractors [10]. It then shows that fluctuations are amplified as the laser approaches saddle-node, period-doubling, and crisis bifurcations [11]. These observations lead to extreme-event physics: low-pass-filtered perturbations can trigger rare high-amplitude periodic excursions whose peak-intensity statistics satisfy a rogue-wave criterion [12]. The ensuing analysis of multistate intermittency demonstrates that the amplitude and cutoff frequency of the injected noise regulate which periodic states are visited and how likely extreme pulses become [13].

A complementary step is to determine whether forcing merely changes the state reached from a given preparation or actually removes competing attractors. Numerical basin mapping addresses this distinction: harmonic control can eliminate selected attractors together with their basins, whereas weak random forcing changes the statistical distribution of initial conditions assigned to each surviving state [14]. Building on earlier attractor-annihilation theory and EDFL experiments [15, 16], this basin-resolved perspective connects observed switching to the control of the underlying phase-space organization.

The review also addresses an essential inferential problem. An irregular time series may be caused by external stochastic forcing, deterministic chaos, hidden variables, or a mixture of all three. Therefore, the study of noisy lasers requires methods capable of detecting deterministic constraints even when standard statistics appear stochastic.

Finally, this body of research has moved from characterization toward function. Noise-controlled multistability can serve as a basis for optical logic, where individual periodic laser states encode logical operations [17]. Reservoir computing further provides a framework for determining whether a noisy, partially observed system can be predicted trajectory by trajectory or only statistically cloned [18]. This work reviews the importance of noise in nonlinear dynamical systems, using the erbium-doped fiber laser as a natural and exceptionally useful example. The EDFL is compelling because it concentrates, within a compact photonic cavity, the central themes of complex dynamics: bifurcations, multistability, stochastic activation, intermittency, extreme events, hidden variables, and information processing. Understanding how noise reshapes its dynamics makes it possible not only to explain rare optical events, but also to transform fluctuations into a controllable resource for sensing, computation, and statistical prediction.

2 Conceptual Background: Multistability, Noise, and Criticality

2.1 Multistability and basin structure

A multistable system possesses more than one attractor for the same set of control parameters. The asymptotic state reached by a trajectory therefore depends on its initial condition. The collection of initial conditions converging to an attractor defines its basin of attraction. Basin boundaries can be smooth or fractal, and their geometry can strongly influence sensitivity to perturbations [2]. In experimental systems, multistability is often detected by varying initial conditions, scanning control parameters in opposite directions, or repeatedly resetting the system.

The importance of multistability is twofold. From a fundamental perspective, it creates a state-space organization in which different long-term behaviors coexist. From an applied perspective, it offers a set of possible dynamical states that can represent memory, logic levels, operating modes, or encoded information. However, it also creates ambiguity: small perturbations may redirect the system to an undesired attractor. Noise-induced hopping is precisely the process through which fluctuations exploit this sensitivity.

In a simplified potential analogy, different stable attractors can be represented as wells separated by barriers. This analogy is useful for intuition but incomplete in non-gradient, driven, and chaotic systems. In such systems, switching may involve unstable periodic orbits, chaotic saddles, riddled or fractal basin boundaries, and transient channels rather than thermal activation over a static energy barrier [1, 2]. The EDFL studies reviewed below are important because they provide experimentally accessible examples of multistate hopping in a nonequilibrium optical oscillator.

2.2 Noise-induced transitions and intermittency

Noise-induced switching occurs when a perturbation displaces a trajectory from one basin into another. The resulting time series may alternate between discrete dynamical regimes, producing intermittency. In a two-state system, the key observables are switching probabilities, residence times, transition rates, and the stationary occupation distribution. In a multistate system, the dynamics can be represented by a transition matrix, symbolic sequence, or state network. The number of accessible states may itself depend on noise amplitude and spectrum.

The interplay between deterministic and random structure can be quantified using entropy measures. If a symbolic sequence contains a single repeated state, its Shannon entropy vanishes. When several states occur with comparable probability, the entropy increases. Entropy alone does not identify the underlying mechanism, but in a controlled experiment it provides a compact descriptor of the complexity of the state sequence [10]. Such coarse-grained representations are especially useful when the underlying intensity signal contains fast oscillations while the relevant physical question concerns the order and probability of visits to attractors.

2.3 Bifurcations and critical susceptibility

A bifurcation is a qualitative change in the set or stability of solutions caused by variation of a control parameter. In the EDFL literature considered here, saddle-node bifurcations create or destroy periodic attractors, period-doubling transitions change temporal periodicity, and crises reorganize chaotic or attracting sets [11]. Close to a bifurcation, restoring stability weakens and the response to perturbations may become large. This is the basis of prebifurcation noise amplification.

Theoretical studies of noisy bifurcations have shown that small perturbations can be amplified near critical points and that the response depends on both deterministic normal forms and noise properties [3, 4]. In applications, this observation motivates early-warning indicators. However, an increase in variance, autocorrelation, or spectral power should not be interpreted as a universal predictor without system-specific validation. Measurement noise, slow parameter drift, nonstationarity, and changes in forcing can imitate critical signals. The EDFL platform is valuable because the control parameter can be scanned reproducibly and the noise input can be adjusted deliberately.

3 Erbium-Doped Fiber Lasers as a Platform for Nonlinear Dynamics

3.1 Physical architecture and technological relevance

An EDFL is based on an optical fiber whose core contains erbium ions. Optical pumping raises ions to an excited manifold, and stimulated emission near the 1.5- μm region supports laser action. Fiber Bragg gratings, mirrors, couplers, wavelength-division multiplexers, isolators, and photodetectors can be incorporated into a compact cavity and measurement chain. Rare-earth-doped fiber lasers offer high gain, single-mode operation, robust beam guidance, and compatibility with telecommunication components [8, 9].

The Fig 1, shows the EDFL configuration used in the 2009 prebifurcation study consisted of a 1560-nm fiber laser with a 3.51-m Fabry-Pérot cavity formed by an 82-cm heavily doped erbium fiber, a Faraday rotating mirror, and a fiber Bragg grating. The cavity was pumped by a 977-nm diode. A wavelength-division multiplexer guided the output to a photodetector, oscilloscope, and Fourier spectrum analyzer [11]. The Faraday rotating mirror was used to reduce polarization-mode-beating effects that could add unwanted complexity. These experimental details matter because they identify the physical source of the observed dynamics and delimit the model's intended regime.

3.2 EDFL Dynamical Model

The pump-modulated erbium-doped fiber laser is described as a class-B laser. Since polarization relaxes much faster than the optical field and the population inversion, its dynamics can be reduced to two coupled equations for the intracavity power $P_g(t)$ and the normalized upper-level population $y(t)$. The model includes gain saturation, ground-state absorption, excited-state absorption, cooperative upconversion, amplified spontaneous emission (ASE), and periodic and stochastic pump forcing [11].

The intracavity power obeys

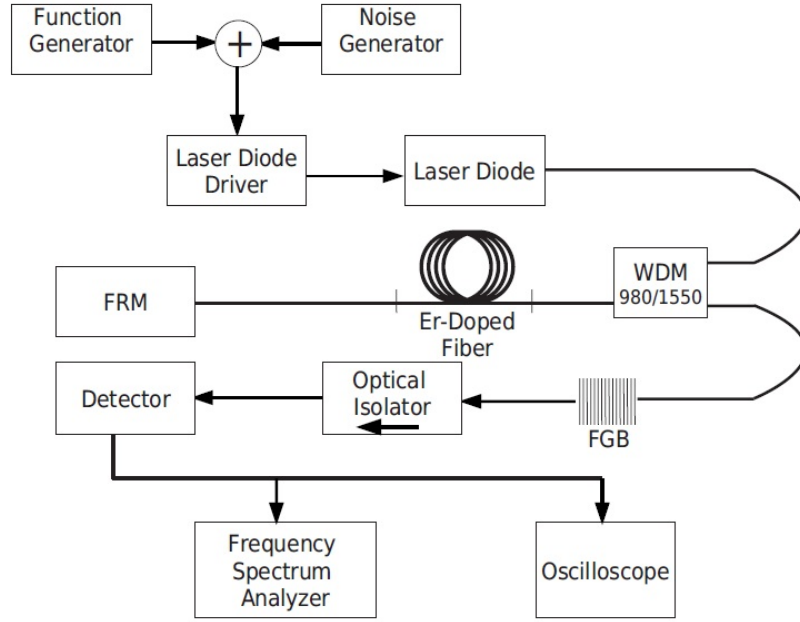


Figure 1: Experimental EDFL setup, [11].

$$\frac{dP_g}{dt} = \frac{2L_0}{T_r} \left[P_g \{ \gamma \alpha_0 [(\tau - \xi)y - 1] - \alpha_l \} + 2\gamma \alpha_0 (\tau - 1)y \frac{\Delta \lambda_g}{\Delta \lambda_{Er}} K(y) \exp(-L\alpha_l) \right], \quad (1)$$

where the first contribution represents the net balance between optical gain and cavity losses, while the second term accounts for ASE seeding of the laser mode.

The upper-level population evolves as

$$\begin{aligned} \frac{dy}{dt} = & -\frac{\gamma \alpha_0 N_0 S_a}{h\nu_g} P_g (\tau y - 1) - \frac{y}{\tau_0} + \frac{4[K'(y) - 1] \gamma \alpha_0 (\tau - 1)}{\tau_0 \Delta \lambda_{Er} N_0 S_a} \\ & - \frac{y^2}{\tau_1} + \frac{[1 - \exp(-\varepsilon \alpha_0 L_0 (1 - y))] S_p}{N_0 L_0 h\nu_p} P_p [1 + A \sin(2\pi f_m t) + n\zeta]. \end{aligned} \quad (2)$$

The terms in Eq. (2) describe, respectively, stimulated transitions, spontaneous relaxation of isolated Er^{3+} ions, ASE-related population depletion, cooperative upconversion in excited ion pairs, and pump absorption. The final factor contains harmonic modulation of amplitude A and frequency f_m , together with externally applied noise of amplitude n ; ζ represents the random sequence.

The ASE coefficients averaged over the active fiber are

$$K(y) = \frac{\exp[L_0 \gamma \alpha_0 ((\tau - \xi)y - 1)] - 1}{L_0 \gamma \alpha_0 ((\tau - \xi)y - 1)}, \quad (3)$$

$$K'(y) = \frac{\exp[L_0 \gamma \alpha'_0 ((\tau' - \xi')y - 1)] - 1}{L_0 \gamma \alpha'_0 ((\tau' - \xi')y - 1)}. \quad (4)$$

For simplicity, the model assumes

$$\tau' = \tau, \quad \xi' = \xi, \quad \gamma' = \gamma. \quad (5)$$

The main definitions are

$$\alpha_0 = N_0 \sigma_{\text{GSA}}, \quad \tau = 1 + \frac{\sigma_e}{\sigma_{\text{GSA}}}, \quad \xi = \frac{\sigma_{\text{ESA}}}{\sigma_{\text{GSA}}}, \quad (6)$$

$$\gamma = 1 - \exp\left(-\frac{S_a}{S_w}\right), \quad S_a = \pi a^2, \quad S_w = \frac{\pi w_0^2}{2}, \quad (7)$$

$$T_r = \frac{2n_0 L}{c}, \quad L = L_0 + l_0, \quad \alpha_l = \alpha - \frac{\ln(R_1 R_2)}{2L}. \quad (8)$$

Here, N_0 is the erbium-ion concentration, σ_{GSA} , σ_e , and σ_{ESA} are the ground-state absorption, emission, and excited-state absorption cross sections, respectively. The quantities τ_0 and τ_1 are the relaxation times of isolated ions and excited ion pairs. The parameters L_0 , l_0 , and L define the active, passive, and total cavity lengths, whereas R_1 and R_2 are the cavity reflectivities.

Including ASE is essential in this model because it permits agreement between simulations and the experimentally measured relaxation oscillation frequency, close to 30 kHz for the operating conditions reported in [11], and its references.

The model contains physically meaningful parameters: photon round-trip time, small-signal absorption, threshold losses, erbium-ion lifetime, overlap between the guided mode and doped core, absorption and stimulated-emission cross sections, and pump absorption. Its strength is that the same basic equations can reproduce experimentally observed relaxation oscillations, periodic locking, multistability, and responses to modulation. Its limitation is that it represents a reduced effective description: spatial, polarization, longitudinal-mode, thermal, and technical noise degrees of freedom may not be fully resolved. The laser parameters for equations 1 and 2, are as the next Table 1:

Table 1: Parameters used in the numerical simulations of the pump-modulated erbium-doped fiber laser (EDFL). Values are reproduced from Table I of Huerta-Cuellar et al. [11].

Symbol	Description	Value	Units
λ_g	Laser-generation wavelength	1560	nm
λ_p	Pump wavelength	977	nm
$h\nu_p$	Pump-photon energy	2.037×10^{-19}	J
α_0	Small-signal absorption coefficient at λ_g	0.06	cm^{-1}
N_0	Er^{3+} ion concentration	2.4×10^{19}	cm^{-3}
σ_{GSA}	Ground-state absorption cross section	2.48×10^{-21}	cm^2
σ_e	Emission cross section	2.25×10^{-21}	cm^2
τ_0	Lifetime of an isolated excited Er^{3+} ion	10	ms
τ_1	Effective relaxation time of excited Er^{3+} - Er^{3+} pair clusters	0.71	ms
$\Delta\lambda_g$	Er^{3+} generation bandwidth	25	nm
$\Delta\lambda_{\text{Er}}$	Er^{3+} emission-band width	10^3	nm
w_0	Guided fundamental-mode radius	3.5	μm
a	Erbium-doped fiber-core radius	2.7	μm
L_0	Active erbium-fiber length	0.82	m
L	Total cavity length	3.51	m
l_0	Total length of passive intracavity fiber components	2.61	m
R_1	Faraday rotating mirror reflectivity	1.00	–
R_2	Fiber Bragg grating reflectivity	0.91	–
ε	Ratio of pump absorption coefficients at λ_p and λ_g	0.7	–
ξ	Ratio of excited-state to ground-state absorption cross sections	0.24	–
η	Ratio of generation to spontaneous-emission bandwidths	2	–
n_0	Refractive index of the fiber core	1.46	–

Note. The parameter names follow the physical definitions given in the model description of Huerta-Cuellar et al. [11]. Verify the symbols against the notation ultimately adopted in the review, since different EDFL articles sometimes use equivalent coefficients with different symbols.

Table 2: Periodic states and their roles in the EDFL narrative.

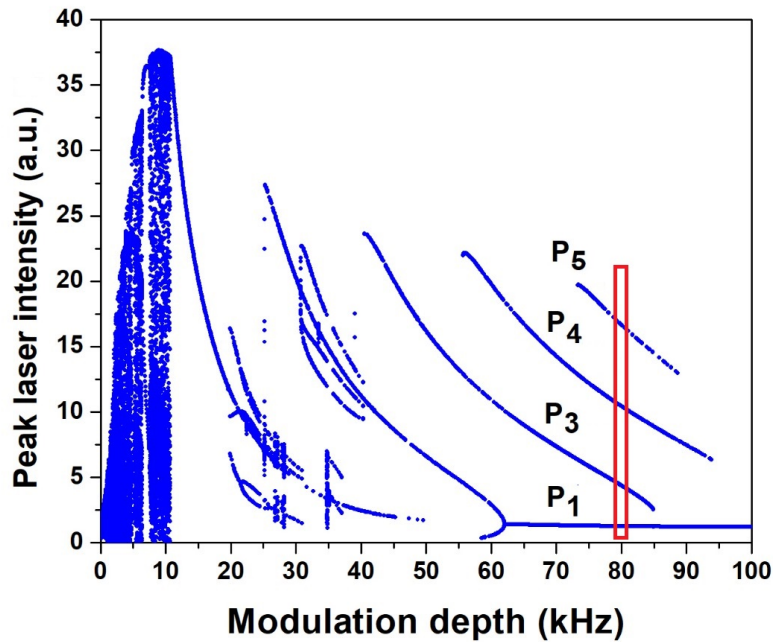
State	Spectral/dynamical interpretation	Role in the reviewed studies
$P1$	Fundamental response, dominant frequency near f_m	Baseline attractor; one endpoint of hopping; associated with OR in the logic proposal
$P3$	Period-three subharmonic response, near $f_m/3$	Coexisting attractor; first state often involved in hopping; associated with AND
$P4$	Period-four subharmonic response, near $f_m/4$	Large-amplitude state; rare occupation can generate rogue-wave-like events; associated with NOR
$P5$	Period-five subharmonic response, near $f_m/5$	Appears in extended multistate intermittency; associated with NAND

The pump can be harmonically modulated as

$$P_p(t) = p [1 - m \sin(2\pi f_m t)], \quad (9)$$

where p denotes the pump level in the absence of modulation, m is modulation depth, and f_m is the driving frequency. Under this forcing, the EDFL can support a fundamental period-one response $P1$ as well as subharmonic periodic responses $P3$, $P4$, and $P5$. The latter have dominant frequencies near $f_m/3$, $f_m/4$, and $f_m/5$, respectively [13, 17].

The numerical bifurcation diagrams, Figure 2 in the 2012 and 2024 studies show parameter regions in which several of these orbits coexist. Basin calculations at fixed modulation frequency reveal that increasing modulation depth changes their relative basin volumes: subharmonic attractors emerge and expand while the relative basin of $P1$ may shrink [13]. This provides a direct dynamical interpretation of why externally injected noise can change the observed state distribution without changing the nominal deterministic control parameters.

**Figure 2:** Bifurcation diagram [17].

4 Experimental Characterization of Hopping Dynamics in a Multistable Fiber Laser

The 2008 study *Experimental characterization of hopping dynamics in a multistable fiber laser* provides the experimental foundation of the review [10]. The central question was how external noise changes the dynamics of a fiber laser operating in a multistable region. At a low noise amplitude, the laser remains on a periodic attractor selected by initial conditions. As the noise increases, the system begins to switch between coexisting periodic states. The observed dynamics progresses from $P3$ – $P1$ hopping to regimes involving $P4$ and ultimately $P5$.

4.1 From state persistence to multistate hopping

At low noise, the experiment showed coexistence of periodic attractors whose characteristic power spectra contained distinct subharmonic components. The system remained in the chosen attractor for arbitrarily long observation times within the experimental window, apart from noisy fluctuations around the periodic orbit. When input noise exceeded a threshold, the laser jumped between $P3$ and $P1$. At larger amplitudes, three and four periodic states participated in the intermittent dynamics [10].

The appearance of $P5$ only at a sufficiently large noise amplitude is particularly informative. It raises the possibility that a state with a small basin is inaccessible in practice without perturbation, or that noise reorganizes the effective dynamics so that a new periodic state becomes observable. The authors appropriately state that the experiment cannot fully distinguish these possibilities because all initial conditions cannot be varied exhaustively. This caution illustrates a recurring challenge in experimental multistability: observed state sets can depend on finite sampling, preparation protocol, and noise level.

4.2 Noise transfer and saturation-like behavior

The relation between input noise N_{in} and output spectral noise N_{out} was measured for individual periodic states and for hopping regimes. For isolated $P1$, $P3$, and $P4$ attractors, the output spectral component was approximately linear in the input amplitude, with larger slopes for higher-period orbits. In contrast, once the laser began hopping among states, the output-noise dependence became much weaker. Figure 3, shows the reported $P3$ – $P1$ – $P4$ – $P5$ hopping regime, the effective slope was substantially smaller than the slope associated with the isolated periodic states [10].

This behavior was described as a noise-saturation effect. Its exact physical mechanism remained unresolved, but the proposed interpretation was that input-noise energy may be redistributed over an increasing number of transitions and states. That interpretation is plausible but should be treated as a hypothesis rather than a settled mechanism. A future experiment could test it by measuring state-resolved spectra, transition-conditioned fluctuations, and the dependence of output power on dwell time in each attractor.

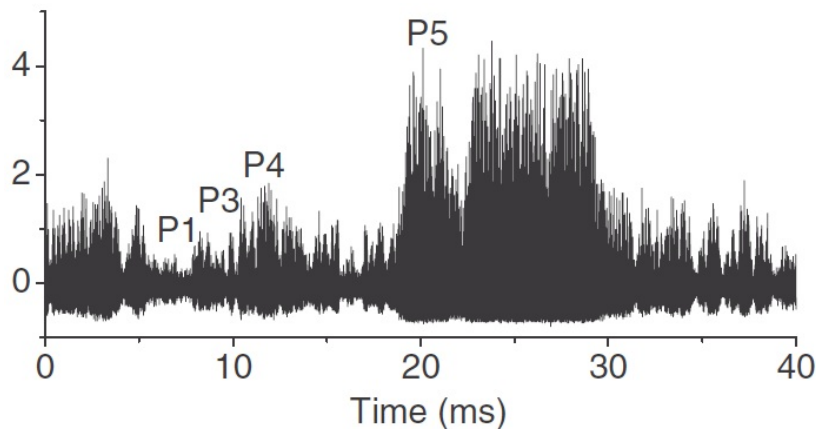


Figure 3: Four attractors interacting in hopping regime, [10].

4.3 Occupation probabilities, entropy, and residence times

The probability of visiting a periodic state is a key observable in noise-driven multistability. In the 2008 experiment, the probability of visiting $P3$ decayed exponentially with noise amplitude in the hopping regime, whereas the

probability of $P1$ increased. The crossover in state probabilities coincided with a change in the output-noise curve, suggesting that occupation redistribution and noise transfer are linked [10, 13].

The authors encoded the sequence of attractor visits symbolically and computed a Shannon entropy rate. A purely periodic regime yields zero entropy because only one symbol appears. The entropy rose when new states entered the dynamics, then remained approximately constant inside a multistate region. Residence-time statistics also displayed scaling behavior, including a reported $-3/2$ power-law feature for a distribution associated with period-one windows in a two-state intermittent regime [10]. These results establish that the relevant dynamics are not captured by amplitude statistics alone: state sequencing and temporal organization matter.

4.4 Connection with general multistable dynamics

The EDFL experiment is consistent with prior observations of noise-induced attractor hopping in multistable systems [1]. More generally, multistability has been identified as an organizing principle in physical, biological, and cognitive systems [2, 19]. The laser contribution is distinctive because the attractors are optical periodic states with well-defined spectral signatures and the noise input can be electronically controlled. This makes the EDFL a promising system for controlled studies of multistate transition networks.

5 Prebifurcation Noise Amplification in a Fiber Laser

The 2009 study *Prebifurcation noise amplification in a fiber laser* investigated how intrinsic and externally added noise behave near critical transitions [11]. The work combined experiment and numerical modeling to address a basic question: can the approach to bifurcation be detected through enhanced fluctuations in an EDFL?

5.1 Bifurcation structure and experimental protocol

The experimental bifurcation diagram was constructed by scanning modulation frequency while recording the peak-to-peak laser intensity. Initial conditions were varied by switching the signal generator on and off and by scanning the modulation frequency in opposite directions. The resulting diagram displayed periodic states and chaotic regions, including $P3$, $P4$, and $P5$ branches born and destroyed through saddle-node bifurcations. Crisis and inverse period-doubling transitions were also identified [11].

This protocol illustrates an important experimental principle: detecting multistability requires intentional perturbation or resetting. A standard one-direction parameter scan may miss coexisting branches. The EDFL therefore serves not only as a source of complex behavior but also as a controlled system in which state preparation can be incorporated into the measurement design.

5.2 Experimental evidence of amplification

The spectral noise density at the modulation frequency was measured as the modulation frequency approached critical points. For both intrinsic laser noise and externally added noise, the noise floor rose near saddle-node, period-doubling, and crisis bifurcations. The closer the operating point was to a critical transition, the stronger the amplification. In the reported data, noise near saddle-node and crisis points could increase by approximately 30 dB, i.e., by about three orders of magnitude in amplitude [11].

The amplification was most pronounced at the corresponding subharmonic frequency of each periodic state. This reflects the dynamical selectivity of the laser: fluctuations are not amplified uniformly at all frequencies but most strongly in the channels associated with the approaching periodic instability. Near a period-doubling bifurcation, amplification was stronger for smaller input-noise levels, consistent with earlier work on small-signal amplification near bifurcations [4].

5.3 Theoretical interpretation and wider context

Numerical simulations using the EDFL model reproduced the experimental rise in spectral noise near saddle-node bifurcations. The fact that the same model also reproduced the relaxation-frequency dependence strengthened the interpretation that the observed effect originates in nonlinear EDFL dynamics rather than in frequency-dependent noise generation by the pump source [11]. Fig. 4 shows an experimental and theoretical comparative of the noise amplification obtained for frequency modulation variation.

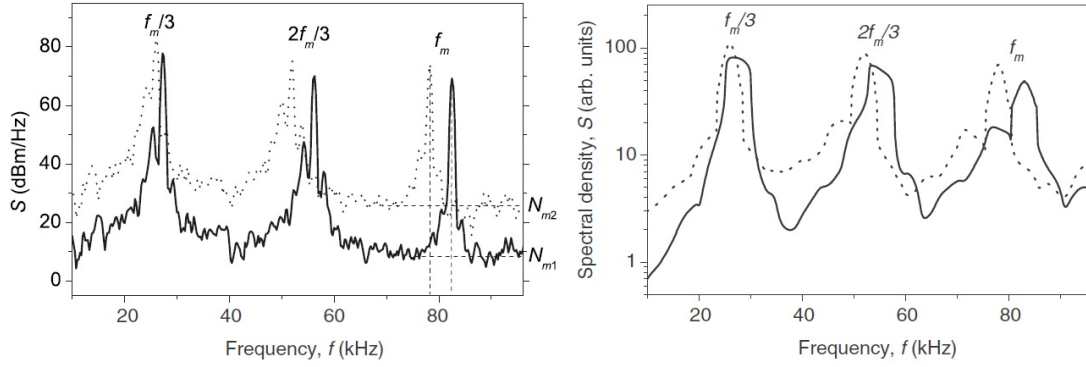


Figure 4: Experimental power spectrum analysis (left) and theoretical (right) comparison of the noise amplification obtained for frequency modulation variation, [11].

The general idea of critical amplification has relevance beyond lasers. Noise growth near bifurcations can be related to critical slowing down, reduced restoring forces, and increased susceptibility. Such behavior has been discussed in nonlinear optical systems, climate variability, population dynamics, and excitable systems [3, 5]. Yet, early-warning applications must be formulated carefully. A fluctuation increase can signal an approaching bifurcation only if the relevant control parameter, forcing statistics, and measurement chain are sufficiently stable. The EDFL provides a valuable testbed for determining which indicators remain reliable under controlled but realistic noise conditions.

6 Control of basins of attraction in a multistable fiber laser

The article *Control of basins of attraction in a multistable fiber laser* adds a global phase-space perspective to the preceding studies of hopping and critical amplification. Rather than asking only which periodic responses appear in a measured trace, Pisarchik and Jaimes-Reategui [14] investigate how their basins change under deterministic and stochastic pump perturbations. The distinction is important: failure to observe an attractor does not establish its destruction, because its basin may be small or inaccessible to the experimental preparation protocol. The new evidence in this study is numerical basin mapping; its agreement with earlier experimental results on harmonic control supports, but should not be confused with, direct experimental measurement of the basins.

6.1 Physical model and connection to earlier control studies

The study builds on the proposal that slow parameter modulation can remove an unwanted attractor through a boundary crisis, rather than simply transfer a trajectory to another state [15]. In an EDFL experiment, additional weak harmonic modulation had already been shown to eliminate one or two stable limit cycles, with the outcome depending on the control frequency and amplitude [16]. These two precedents are references 13 and 26, respectively, in the basin-control article. The subsequent basin calculations examine the phase-space organization behind that experimentally observed selection.

The numerical description uses two rate equations for the intracavity radiation variable P and the longitudinally averaged upper-level population N , derived from an erbium-level scheme that includes excited-state absorption and spontaneous emission into the fundamental laser mode, as shown in Figure 5 [14]. Its modeling antecedent, reference 27 of that article, connected pump-modulated EDFL dynamics with coexisting periodic and chaotic responses through experimental and theoretical bifurcation diagrams [20]. Importantly, the basin-control model explicitly excludes mechanisms for autonomous self-pulsing; its periodic responses are generated through pump modulation [14]. It should therefore not be treated as an interchangeable transcription of the more elaborate model used in the separate 2009 prebifurcation study.

In the article's normalization, the unmodulated pump parameter is $p = 7.4 \times 10^{19} \text{ s}^{-1}$, and the relaxation-oscillation frequency is approximately 30 kHz. The principal basin calculations use driving frequency $f_d = 70.2 \text{ kHz}$ and modulation depth $m_d = 0.8$, where the responses $P1$, $P3$, and $P4$ coexist [14]. These values specify a particular operating point, not universal thresholds for EDFLs. In particular, the pump parameter expressed in s^{-1} should not be inserted as a power in watts into a differently normalized model.

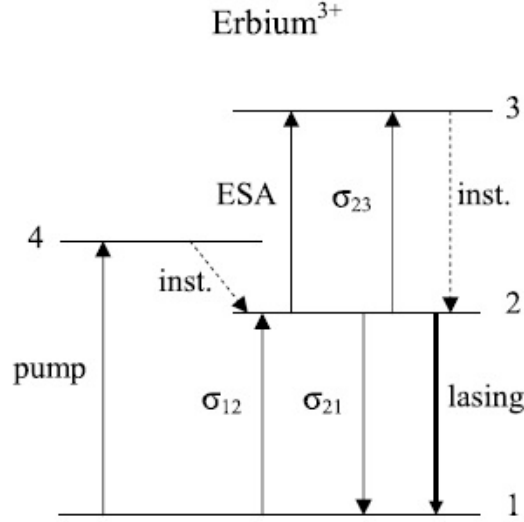


Figure 5: Energy Erbium diagram, [14].

6.2 Harmonic control and destruction of competing basins

The control protocol introduces a second sinusoidal signal into the depth of the principal pump modulation. Using the notation of the source, the resulting pump is

$$P_p(t) = p \left\{ 1 - \left[m_d^0 - m_c \sin(2\pi f_c t) \right] \sin(2\pi f_d t) \right\}, \quad (10)$$

where m_d^0 is the baseline modulation depth and m_c and f_c are the amplitude and frequency of the control signal [14, Eq. 6]. Thus, the control modulates the driving amplitude rather than merely adding a second independent pump tone. This distinction matters when reproducing the excitation electronically or numerically.

As the control is increased, the driven responses change their extent and structure in phase space. The authors interpret attractor annihilation through the development of a two-frequency response, possible further bifurcations, and eventual collision with a basin boundary. Control-frequency dependence is pronounced: the smallest amplitudes required for annihilation occur below the relevant resonance frequency, consistent with a nonlinear rather than purely linear response [14]. The earlier theoretical work provides the underlying boundary-crisis interpretation, while the EDFL basin maps make the associated loss of accessible initial conditions explicit [15, 14].

Two reported parameter sets illustrate the outcome. At $m_c = 0.128$ and $f_c = 2.6$ kHz, the P_4 attractor and its basin disappear while P_1 and P_3 remain. At $m_c = 0.38$ and $f_c = 2.8$ kHz, both higher-period responses are removed, leaving P_1 throughout the sampled initial-condition domain [14, Fig. 6(c, d)]. The significance of these maps is that selection is not demonstrated only for one trajectory: many initial conditions are tested after applying control. Nevertheless, finite numerical basin sampling does not constitute an exhaustive mathematical proof over an unbounded phase space.

6.3 Weak noise and probabilistic basin assignment

The stochastic protocol is different from the harmonic control above. The pump is written as

$$P_p(t) = p \left[1 - m_d \sin(2\pi f_d t) + \eta \zeta(t) \right], \quad (11)$$

where η controls the perturbation amplitude and the numerical random variable satisfies $\zeta \in [-1, 1]$ [14, Eq. 7]. The source describes this forcing as white noise. A reproduction should preserve its implemented sampling and bandwidth rather than silently substitute a Gaussian Wiener increment, whose normalization would require an additional specification.

The authors focus on weak forcing, described as $\eta < 0.1$, for which sustained intermittency is not observed under their simulation conditions; their statistical illustration also includes the boundary value $\eta = 0.1$ [14]. In this regime, noise can redirect a transient toward a different final periodic response without producing repeated hopping after that response is reached. Consequently, the object being measured is the distribution of outcomes across repeated preparations, not the residence-time distribution along a single intermittent trajectory.

Table 3: Distinct control outcomes in the basin study. Conditions and results are taken from Pisarchik and Jaimes-Reategui [14]; the baseline is $m_d^0 = 0.8$ and $f_d = 70.2$ kHz.

Protocol	Reported condition	Outcome and evidential scope
No additional control	$m_c = 0, \eta = 0$	Coexisting $P1$, $P3$, and $P4$ with distinct basins.
Harmonic control	$m_c = 0.128, f_c = 2.6$ kHz	$P4$ and its basin disappear in the numerical map.
Harmonic control	$m_c = 0.38, f_c = 2.8$ kHz	Only $P1$ remains in the sampled domain.
Weak stochastic forcing	Low-noise regime described by $\eta < 0.1$	Outcome distributions change across repeated preparations without sustained hopping in the reported regime.

For each noise amplitude, the basin calculation is repeated 100 times over the initial-condition region shown in the source's Fig. 6. Each repetition yields a count of points assigned to each periodic state, and the resulting ensemble defines a distribution of basin-point counts [14, Fig. 7]. The method can be summarized, using notation introduced here for clarity, as

$$B_i^{(r)}(\eta) = \sum_{j=1}^M \mathbf{1}\{\mathcal{C}_r(\mathbf{x}_{0,j}; \eta) = i\}, \quad i \in \{1, 3, 4\}, \quad (12)$$

where M is the number of sampled initial conditions, r indexes a repeated noisy basin calculation, and $\mathcal{C}_r$ denotes classification after the transient. Equation (12) restates the counting protocol; it is not an additional laser evolution equation or a fitted result. For a fixed uniformly sampled domain, $B_i^{(r)}/M$ is a sampled basin fraction, whose numerical value depends on the domain and classification protocol.

6.4 Statistical findings and limits of the resonance interpretation

The distributions change systematically with noise amplitude. Their most probable basin-point counts generally increase for $P1$ and $P3$ and decrease for $P4$, approximately linearly over the explored range [14]. At the same time, the probabilities associated with particular counts vary nonmonotonically. For example, at $\eta = 0.1$ the paper reports a probability of approximately 30% for a $P1$ count near 1.3×10^3 , compared with approximately 10% for a count near 1.9×10^3 [14]. These are probabilities of obtaining specified counts across repeated basin maps; they are not probabilities that a single laser trace spends 30% or 10% of its time in $P1$.

The authors describe maxima and minima in the probability distributions as resonance-like and suggest a possible connection with stochastic resonance [14]. This wording should remain qualified. Their reported evidence concerns the statistics of basin assignment, rather than a demonstrated optimum in signal-to-noise ratio or a measured matching of transition times to a periodic signal. Accordingly, the result establishes nonmonotonic statistical sensitivity to noise, while a specific stochastic-resonance mechanism would require further diagnostics.

6.5 Connection to extreme events and information processing

This study provides a useful distinction between three operations: removing a competing attractor, biasing the response selected after a transient, and sustaining transitions among surviving states. Harmonic annihilation concerns the first; the weak-noise basin analysis concerns the second; the hopping and multistate-intermittency studies concern the third [14, 10, 13]. A change in output-state statistics should therefore not automatically be interpreted as attractor destruction.

The same distinction clarifies the transition to extreme-event physics. The presence of a high-amplitude branch and the accessibility of its basin help organize possible excursions, but the 2009 basin study does not itself establish rogue-wave statistics; that evidence belongs to the subsequent extreme-pulse studies [12, 13]. Likewise, probabilistic basin selection provides a conceptual precursor to state-selective optical logic, not a demonstration of a working logic gate [17]. For future devices, the combined results suggest comparing the reliability of deterministic elimination of unwanted states with deliberate probabilistic selection, while measuring reset dependence, switching latency, and output errors. This last comparison is a research direction motivated by the reviewed results rather than a performance claim made by the basin-control article. Figure 6 shows four stages, from all present attractors to the annihilation of the less probable ones, in the system controlled on the $P1$ attractor.

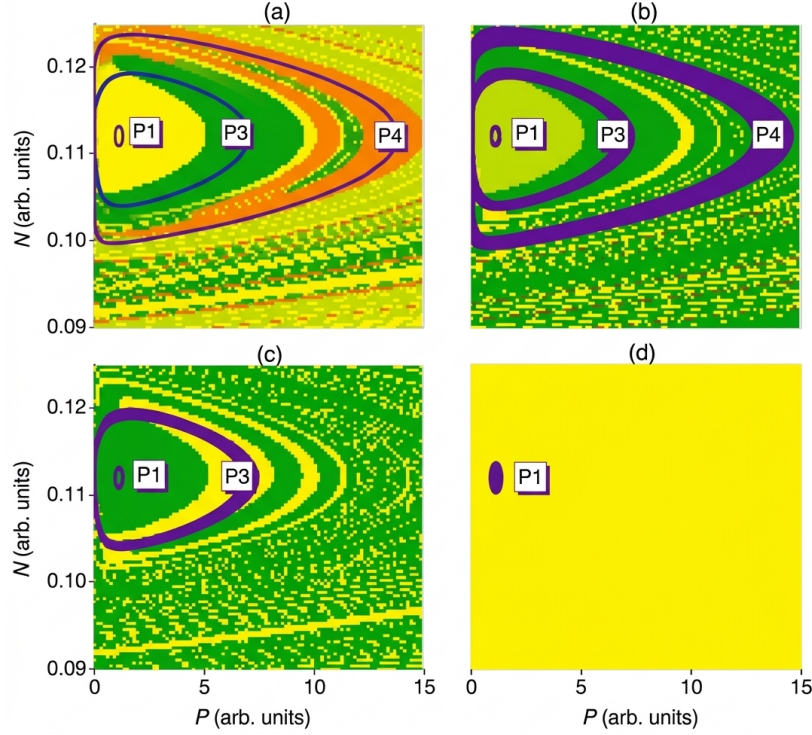


Figure 6: Annihilation of P3 and P4 attractors, [14].

7 Rogue Waves in a Multistable System

7.1 Extreme events in optics

The term “rogue wave” refers to a rare event of exceptional amplitude whose probability is much larger than expected from a simple Gaussian background model. The concept originates in ocean-wave science but has become important in optics after the observation of extreme pulses in nonlinear fiber systems [21]. In optical contexts, several mechanisms may generate extreme events, including modulation instability, nonlinear focusing, soliton interactions, chaotic dynamics, and state switching. The EDFL work reviewed here identifies a mechanism centered on multistability and noise-induced transitions [12].

7.2 Rare excursions to high-amplitude periodic states

In *Rogue Waves in a Multistable System*, the EDFL was operated in a region where $P1$, $P3$, and $P4$ coexist. The pump current received both harmonic modulation and low-pass-filtered noise. A rare event was observed when the laser left the ordinary $P1$ response and briefly entered the large-amplitude $P4$ state for several periods before returning to $P1$. The average time between events was reported to be about 30 s under the explored conditions [12].

This observation provides a physically specific interpretation of a rogue pulse. The event is not an isolated amplitude fluctuation around a single attractor; it is a transient visit to a different, coexisting dynamical regime. Such a mechanism makes the event probability controllable through the noise parameters and the underlying basin geometry. It also implies that a large-amplitude event can have a recognizable precursor in terms of state-space movement, even if that precursor is difficult to infer from a scalar intensity measurement alone.

7.3 Statistical evidence and non-Gaussian tails

The rogue-wave interpretation was supported by probability density functions of peak intensity. The large peaks appeared far more often than expected under Gaussian statistics, producing a heavy-tail signature [12], Fig. 7. This statistical criterion is essential because visual inspection alone cannot establish whether a high pulse is genuinely anomalous relative to the background distribution.

In numerical simulations, the probability of a calculated period-four rogue event was very small, and the probability maps in noise-amplitude–cutoff-frequency space showed that high-amplitude states became more likely

as the noise cutoff approached the relaxation frequency. The experimental and numerical results thus agree on the key point: extreme events emerge from an interplay between stochastic modulation and deterministic nonlinear multistability.

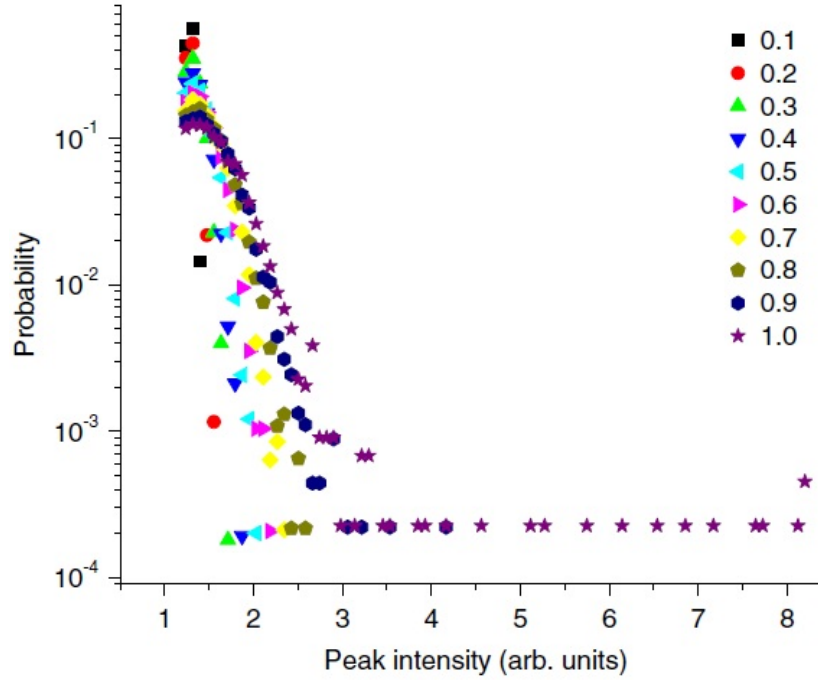


Figure 7: Non-Gaussian heavy-tail rogue-wave signature in numerical results, [12].

7.4 Relation to other rogue-wave mechanisms

The EDFL mechanism should be placed in the broader rogue-wave literature without conflating distinct processes. In ocean models, nonlinear wave interactions and modulation instability can create large transient waves [22, 23]. In optical-fiber experiments, nonlinear propagation and soliton dynamics can produce extreme pulses [21]. In the EDFL case, the central mechanism is noise-assisted switching among coexisting attractors. These mechanisms may share statistical signatures but differ in their dynamical origin. A major contribution of the EDFL studies is to show that multistability provides a concrete route to heavy-tailed pulse statistics.

8 Multistate Intermittency and Extreme Pulses in a Fiber Laser

The 2012 article *Multistate intermittency and extreme pulses in a fiber laser* extended the rogue-wave study by examining how the amplitude and spectral content of noise regulate the occupancy of periodic states [13]. The work is central to the present review because it converts the qualitative claim that “noise matters” into a parameter-dependent control map.

8.1 Stochastic modulation

The pump modulation was represented as

$$P_p(t) = p [1 - m_d \sin(2\pi f_d t) + \eta G(\zeta, f_n)], \quad (13)$$

where η is the noise amplitude, G is a zero-mean random function, ζ denotes the random input, and f_n is the low-pass cutoff frequency. White noise was filtered with a discrete low-pass Butterworth filter. This formulation emphasizes that the perturbation has at least two physically distinct controls: intensity and bandwidth.

For small noise amplitude, the initially selected P_1 attractor remained stable while other attractors could coexist. For larger amplitude, the laser entered intermittent regimes. Depending on the noise parameters, it switched between P_1 and P_3 , among P_1 , P_3 , and P_4 , or among all four periodic states. The period-five response was notable because it

appeared only in the intermittent regime under the explored conditions, indicating that noise can make otherwise inaccessible or unstable dynamics observable [13].

8.2 Spectral selectivity of noise-induced switching

The major result is that the low-frequency content of noise controls access to high-amplitude subharmonic responses. In the numerical analysis, P_4 occurred only when the cutoff frequency was below 1 MHz and P_5 only below 200 kHz. The boundaries of intermittency regimes had minima close to the relaxation frequency of the EDFL, around 30 kHz [13]. This reveals a resonance-like susceptibility: filtered noise is especially efficient when it contains temporal components compatible with the laser's intrinsic dynamics.

The result is conceptually important because it rejects a simple variance-based view of stochastic forcing. Two noise signals with the same amplitude can have qualitatively different effects if their correlation time or spectral content differs. In a nonlinear oscillator, the temporal organization of fluctuations can determine whether they merely perturb a periodic orbit or drive the system across basin boundaries.

8.3 Extreme pulses as rare state occupation

The probabilities of P_4 and P_5 pulses increased as the cutoff frequency decreased, while the probability of P_1 decreased. However, high amplitude alone does not define an extreme event. The authors stressed that such pulses should be treated as extremes only within a region where their occurrence probability is low [13]. This is a rigorous and useful distinction: changing noise parameters can turn a formerly rare event into a common state, at which point it no longer represents a statistical extreme even though its amplitude remains large.

The paper therefore supplies a blueprint for extreme-event engineering. One can map the parameter region where large pulses are both accessible and rare, then tune the noise spectrum to increase or suppress their probability. This framework is potentially relevant to the design of rare-pulse sources, to the avoidance of dangerous excursions in laser systems, and to the general study of extremes in multistable nonlinear systems.

9 Harnessing Multistability: A Novel Approach to Optical Logic Gate Construction Using Erbium-Doped Fiber Lasers

The 2024 article *Harnessing Multistability: A Novel Approach to Optical Logic Gate Construction Using Erbium-Doped Fiber Lasers* moves from analysis to application [17]. Its central proposal is to use noise-induced multistate intermittency as a mechanism for selecting logical operations.

9.1 Dynamical logic and chaos computing

Dynamical systems can implement logic if distinct attractors or responses are assigned to output states. This idea has been developed in chaos computing and nonlinear circuit research [24, 25, 26]. Systems with coexisting attractors are attractive because they can support multiple functional states within one physical device. Logic gates and memory elements have been demonstrated or proposed in nonlinear circuits, cavity magnonics, and optical systems [27, 28, 29].

The EDFL proposal differs from schemes based exclusively on deterministic digital forcing. Here, the laser's noise-controlled intermittent state determines which logic operation is active. The four coexisting periodic states are associated with four logic functions: P_1 with OR, P_3 with AND, P_4 with NOR, and P_5 with NAND [17].

9.2 Noise-controlled probability balancing

The article uses the normalized EDFL model to locate a multistable region where P_1 , P_3 , P_4 , and P_5 coexist near a modulation frequency of 80 kHz. With low-pass-filtered noise, the laser undergoes a progression from a single state to two-state, three-state, and four-state intermittency. The occurrence probability of each period was mapped over noise intensity and filter-frequency parameters [17].

A critical design objective is to find regions in which the probabilities of the periodic states are comparable. If one state dominates, then the corresponding logic operation would be selected disproportionately often. More balanced occupation makes it possible to generate less biased state sequences and to use the local maxima of the laser signal as a dynamical selector of logic operations. DFA was employed to identify correlated regimes and to characterize the temporal organization of the intermittent signal [17, 30].

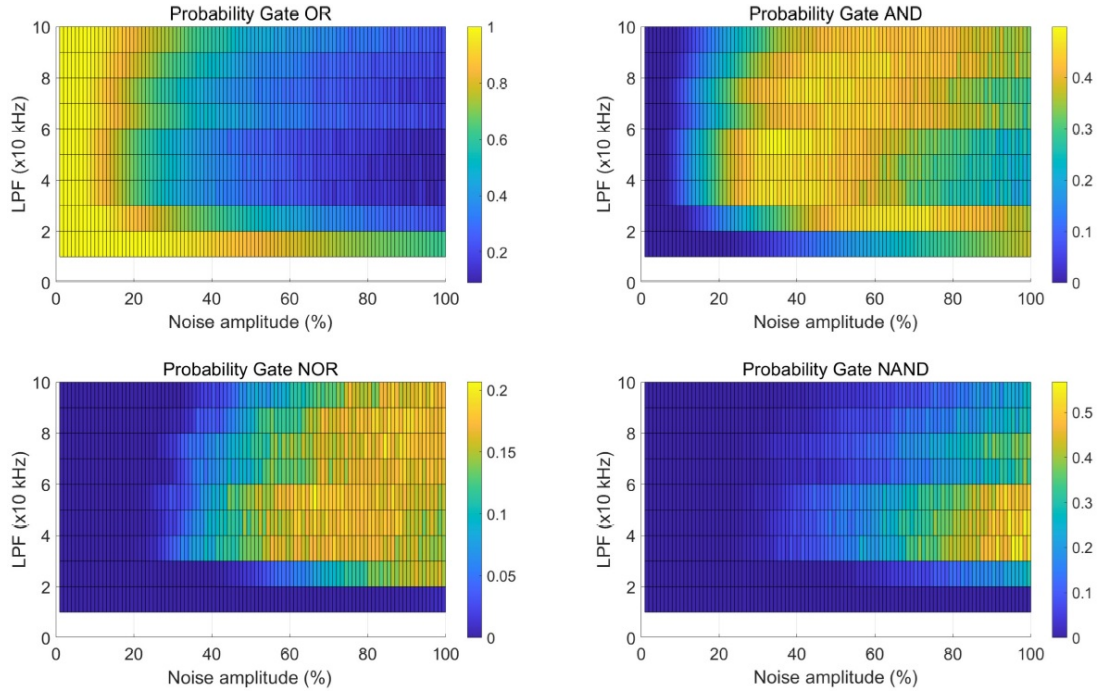


Figure 8: Hopping multistate intermittency regime as a function of noise intensity $\eta \times 10^2$ and a low pass filter for frequency $f_n \times 10$ kHz, [17].

9.3 Potential applications and limitations

The proposal includes an experimental implementation concept involving a function generator, noise generator, laser driver, EDFL cavity, photodetector, and data-acquisition system. The state-dependent amplitude of the laser output would select the logical operation performed on two digital inputs. Beyond logic gates, the authors point to potential use in pseudo-random number generation, encryption, simulations, and optimization [17].

A rigorous review must separate promise from demonstrated performance. The study is numerical and proposes an experimental configuration. It does not yet establish gate speed, error rate, cascaded operation, energy consumption, or robustness to uncontrolled intrinsic noise. These are important next steps. Nevertheless, the work demonstrates a general principle: multistable photonic dynamics can be functionalized when noise is treated as a tunable state-selection input rather than as an uncontrolled disturbance.

10 Stochastic Cloning of Dynamical Systems with Hidden Variables

The 2025 study *Stochastic cloning of dynamical systems with hidden variables* introduces a data-driven perspective on noisy nonlinear systems [18]. It uses reservoir computing to reconstruct the response of stochastic systems driven by known external noise while only part of the internal dynamics is observed.

10.1 Strong versus weak cloning

A strong clone reproduces the trajectory of the target system under the same external noise realization. A weak clone reproduces selected statistical properties, for example distributions of activation intervals, without requiring pointwise trajectory agreement. The distinction is fundamental. Strong cloning requires a deterministic functional mapping from the external input to the observed response. If the same forcing applied to identical systems with different initial conditions does not make them converge, then the response is not uniquely determined by the forcing and exact trajectory cloning is impossible [18].

The study formalizes weak cloning through agreement of moments or relevant statistical metrics between the true and reconstructed processes. This viewpoint is appropriate for stochastic or partially observed systems, where a probabilistic model can be scientifically useful even if it cannot predict individual future events exactly.

Table 4: Noise functions, observables, and applications in the reviewed EDFL studies.

Role of noise	Primary observables	Consequence or application
Perturbation	Spectral background, amplitude fluctuations	Stability limitation; baseline characterization
Critical probe	Spectral noise near bifurcations	Detection of enhanced susceptibility and approaching transitions
Switching mechanism	State probabilities, residence times, transition sequences	Noise-induced attractor hopping and multistate intermittency
Extreme-event regulator	Peak-intensity PDFs, waiting times, probability maps	Control or suppression of rare high-amplitude pulses
Information resource	Periodic-state labels, logic outputs, statistical metrics	Optical logic, pseudo-random sequences, weak stochastic cloning

10.2 Reservoir-computing architecture

The reservoir receives the external noise, the measured system response, and delayed copies of both. A recurrent hidden layer transforms the inputs nonlinearly, and a trained readout produces the predicted signal. Delay-coordinate embedding enriches the representation of partially observed dynamics [18]. Reservoir computing is well suited to nonlinear temporal systems because it can approximate complex input-output mappings without explicitly estimating the full governing equations [31, 32].

Previous work has shown that RC can infer unmeasured variables in chaotic systems, forecast large spatiotemporally chaotic dynamics, construct data-driven digital twins, and predict critical transitions [33, 34, 35, 36]. The stochastic-cloning framework adds a useful theoretical criterion: it identifies when trajectory-level prediction is physically feasible and when only statistical reproduction should be expected.

10.3 Application to the stochastic EDFL

The article examined a stochastic EDFL model with laser intensity and population inversion as state variables. Only the externally injected noise and laser intensity were supplied to the RC; inversion was treated as hidden. Exact prediction was not achieved. The best strong-prediction accuracy remained limited even after adding delayed inputs. Nevertheless, the distribution of interspike activation intervals of the RC output closely matched the distribution for the original EDFL signal, yielding high weak-cloning performance [18].

The authors then performed an experimental verification with a noise-driven EDFL. The weak clone reproduced key statistical properties, although background measurement noise limited the reconstruction quality. This result gives a realistic perspective on data-driven modeling of noisy lasers. A model can be useful for predicting distributions, activation statistics, or risk measures even when the exact pulse sequence is not reproducible.

11 Synthesis: A Hierarchy of Noise Functions in Multistable EDFLs

The studies reviewed above support a hierarchy of roles for noise in EDFLs. First, noise is a *perturbation*: it broadens signals and can destabilize a desired operating state. Second, it is a *probe*: near bifurcations, amplified fluctuations reveal increasing susceptibility. Third, it is a *transition mechanism*: it enables hopping across basin boundaries and creates multistate intermittency. Fourth, it is an *extreme-event control*: through amplitude and filtering, it regulates the probability of rare high-amplitude excursions. Fifth, it is an *information resource*: state occupancy can select logic operations and drive statistical inference.

This hierarchy also identifies experimental requirements. A credible noise-control study must specify the injection point, amplitude calibration, spectral filtering, probability distribution, sampling rate, measurement bandwidth, initial-condition protocol, state-classification method, and observation duration. Without this information, it is difficult to distinguish a true change in nonlinear dynamics from a change in measurement conditions or forcing statistics.

12 Open Problems and Future Directions

12.1 Transition matrices and basin-resolved experiments

Future work should construct state-to-state transition matrices as functions of noise amplitude and correlation time. Such matrices would quantify whether transitions are symmetric, whether particular states act as hubs, and whether direct transitions are suppressed by phase-space structure. Combining these measurements with numerical basin maps and unstable invariant-set calculations would clarify the mechanisms behind hopping and saturation-like noise transfer.

The 2009 basin calculations also motivate a separate repeated-preparation experiment at noise levels below the observed hopping regime [14]. Such a test would estimate the probability of selecting each response following a controlled reset and compare it with finite-time occupation probabilities measured after selection. Because population inversion is not directly initialized or observed in the original experiments, agreement with a simulated basin map must account for the experimentally accessible preparation ensemble rather than assume uniform sampling of the full model state space.

12.2 Early-warning indicators in photonic systems

The observed prebifurcation amplification motivates a systematic comparison of candidate early-warning signals: spectral background, variance, autocorrelation, recovery time, residence-time changes, and symbolic entropy. EDFLs could serve as benchmark systems because their control parameters and noise injection can be manipulated reproducibly. The objective should not be to identify a universal indicator, but to establish operating conditions under which a given observable provides reliable warning of a specified transition.

12.3 Extreme-event forecasting and control

The multistable rogue-wave mechanism suggests that extreme events may be forecast probabilistically from proximity to basin boundaries and state-occupation statistics. A practical research program would measure conditional probabilities of high-amplitude excursions given recent dwell times, spectral fluctuations, or filtered-noise history. Reservoir computing and physics-informed models could be compared for this task. The relevant metric is likely not pointwise prediction but calibrated event probability and false-alarm rate.

12.4 Experimental optical logic

The logic proposal needs experimental validation under controlled and uncontrolled noise. Important metrics include gate-selection error, latency, repeatability, state-recognition robustness, effect of finite detector bandwidth, energy cost, and compatibility with cascaded photonic circuits. An intermediate demonstration could implement a single state-selected operation with repeated trials and full uncertainty analysis before moving to four-gate functionality.

12.5 Hybrid physics–machine-learning models

The stochastic-cloning results motivate hybrid approaches in which a reduced EDFL model supplies physically meaningful coordinates while a reservoir or other learning system captures unresolved effects. Such models may improve statistical prediction without overstating trajectory-level predictability. They could also identify which additional measurements—for example pump-current fluctuations, polarization information, or spectral components—most improve observability of the hidden population-inversion dynamics.

13 Conclusions

The reviewed works establish the diode-pumped erbium-doped fiber laser as a versatile laboratory for the study of noise in nonlinear dynamical systems. The EDFL supports multistability among periodic states, and this deterministic structure allows external noise to produce organized effects: attractor hopping, changes in state probability, enhanced fluctuations near bifurcations, and rare high-amplitude excursions with rogue-wave statistics. The response depends not only on noise amplitude but also on its spectral content, especially relative to the intrinsic relaxation time scale of the laser.

Basin-resolved control supplies the connection between state accessibility and state elimination. Harmonic modulation can remove competing attractors together with their sampled basins, whereas weak stochastic modulation can bias preparation outcomes without sustained intermittency in the reported regime [14]. Preserving this distinction prevents a change in observed state probability from being mistaken for a change in the number of underlying attractors, and helps frame both extreme-event control and reliable photonic state selection.

The review also emphasizes a methodological lesson. Brownian-like spectra, DFA exponents, and heavy-tailed distributions do not uniquely establish stochastic origin. Deterministic systems can exhibit similar signatures, so physically meaningful interpretation requires controlled forcing, validated models, symbolic diagnostics, and carefully designed statistical tests. Ordinal-pattern analysis and reservoir-based stochastic cloning provide complementary tools for this purpose.

Finally, multistability and noise can be exploited rather than merely tolerated. Noise-controlled occupation of coexisting EDFL states offers a route toward reconfigurable optical logic. When exact trajectories are not predictable because of hidden variables and stochastic influences, reservoir computing can still provide weak clones that reproduce relevant statistics. Taken together, these results show that noise is not simply an imperfection in nonlinear photonics. Properly characterized and controlled, it is a tunable probe of criticality, a mechanism for state selection, a driver of rare events, and a resource for photonic information processing.

Declaration of Generative AI and AI-Assisted Technologies in the Manuscript Preparation Process

During the preparation of this review, the authors used Perplexity AI to assist in organizing and revising portions of the text, formatting, preparing content in LaTeX, and verifying references. The authors critically reviewed and edited the material processed with the aid of AI, ensuring that all scientific claims, equations, numerical values, and bibliographic references were consistent with the sources. The authors assume full responsibility for the accuracy, integrity, and final content of the manuscript.

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