

Physics-Informed Neural Network for Sparse-Sensor Inverse Heat Conduction

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Abstract

This study develops a sparse-sensor physics-informed neural network (PINN) to reconstruct a transient two-dimensional thermal field and identify thermal diffusivity, volumetric heat loss, and source amplitude from noisy point measurements. Sixteen interior sensors provide 5424 observations for a moving-source benchmark. The nominal model achieves a mean root-mean-square error (RMSE) of 3.52×10^{-3} normalized temperature units and a global relative L_2 error of 2.87×10^{-2} . The corresponding relative errors in the three identified coefficients are 5.04%, 24.54%, and 7.90%. Under matched data, architecture, initialization, and iteration budget, the physics-informed model reduces mean RMSE by 96.03% relative to a conventional data-only neural network and by 95.56% relative to spatiotemporal interpolation. One-factor-at-a-time tests indicate that spatial sensor scarcity and loss weighting are more influential than temporal subsampling over the ranges examined. The results show that governing-equation constraints provide strong regularization for sparse thermal reconstruction; however, the sensitivity results are diagnostic rather than statistical, and experimental validation remains necessary before deployment.

Keywords— Physics-informed neural network, Inverse heat conduction, Sparse sensors, Thermal field reconstruction, Parameter identification, Sensitivity analysis

1 Introduction

Inverse heat-conduction problems arise in thermal monitoring, digital twins, advanced manufacturing, electronics cooling, and energy systems. In these applications, complete internal temperature fields and effective material properties are seldom measured directly. Classical inverse workflows couple a numerical solver with explicit regularization and an outer optimization loop, but their computational cost and ill-conditioning grow when measurements are sparse, noisy, or confined to a small part of the domain. Scientific machine learning offers a complementary strategy in which observations and governing equations are incorporated into one trainable model [1, 2, 3, 4]. Related data-driven studies have also demonstrated neural-network-based system identification under complex dynamics and noisy observations, including approaches that incorporate probabilistic information to improve modeling accuracy [5].

A physics-informed neural network (PINN) is trained not only from data, but also by penalizing violations of a partial differential equation (PDE), its boundary conditions, and its initial condition [2]. This approach builds on residual and variational neural solvers such as the Deep Ritz method and the Deep Galerkin method (DGM) [6, 7]. Later work has addressed DeepXDE (deep learning for differential equations) and the Scientific Artificial Neural Networks (SciANN) interface [8, 9], gradient imbalance in composite objectives [10], adaptive activations [11], hp -variational physics-informed neural networks (hp -VPINNs), in which h and p denote subdomain scale and polynomial degree, for domain decomposition [12], Bayesian physics-informed neural networks (B-PINNs) [13], and deep operator networks (DeepONets) [14]. Together, these studies show that PINNs are flexible inverse solvers, while also identifying optimization, loss scaling, and uncertainty as persistent limitations [3, 15].

Thermal applications include direct heat-equation solutions, inverse coefficient or heat-flux estimation, manufacturing surrogates, and virtual thermal sensing [4, 16, 17, 18, 19, 20, 21]. Zobeiry and Humfeld studied one- and two-dimensional conduction models with physics-based feature engineering. Oommen and Srinivasan considered data-sparse inverse heat transfer without a conventional surrogate, while He *et al.* and Billah *et al.* examined direct and inverse conduction in materials. Go *et al.* developed a PINN-based virtual thermal sensor. More recent studies

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have investigated improved PINN formulations for transient nonlinear and inhomogeneous heat conduction [22], uncertainty quantification and enhanced formulations for inverse heat-conduction problems [23, 24], and transient thermal problems involving moving heat sources [25]. These studies establish the feasibility of physics-informed learning for inverse and transient thermal problems, but their domains, measurements, unknowns, and error normalizations differ. A direct ranking of their reported errors would therefore be misleading.

A controlled comparison on a single benchmark helps isolate the contribution of physics constraints. In particular, neural identification studies show that model architecture and probabilistic treatment can affect performance under noisy, complex data [5], while recent sequence-based neural models have used learned representations to identify nonlinear robotic systems from heterogeneous measurements [26]. These results motivate careful control of model capacity, available observations, and training conditions when assessing a physics-informed model. They also reinforce the importance of distinguishing improvements associated with the learning architecture from those attributable to the incorporation of physical knowledge.

Another distinction matters for inverse thermal problems: field reconstruction and parameter identification are not necessarily equivalent objectives. A model may reconstruct the temperature field accurately while still exhibiting comparatively large errors in one or more physical coefficients. This is particularly relevant for sparse-sensor problems, where different parameters can produce partially correlated effects on the thermal dynamics. The present study therefore evaluates field reconstruction and parameter recovery separately.

The present study focuses on a controlled sparse-sensor benchmark for transient two-dimensional inverse heat conduction. It reconstructs the thermal field while simultaneously identifying three positive physical coefficients from sixteen noisy interior sensors. The proposed PINN is compared with a data-only neural network using the same architecture and observations, as well as with noniterative spatiotemporal interpolation. The study also examines sensor count, temporal sampling, measurement noise, and individual loss weights through controlled sensitivity runs.

The comparison with existing literature is summarized in Table 1. These studies differ in their physical settings, estimated quantities, measurement configurations, and reported accuracy. Their error normalizations are not directly comparable, so a ranking of their reported errors would be misleading, and a controlled comparison on one benchmark is still needed to isolate the contribution of the physics constraints. The closest prior work, Bidou *et al.* [21], addresses two-dimensional transient inverse conduction with a Gaussian internal source and PINN-guided sensor placement, but treats orthotropic property identification and source identification as separate tasks. None of the cited studies simultaneously recovers the thermal diffusivity, a volumetric heat-loss coefficient, and the source amplitude in a two-dimensional transient problem with a composite moving and stationary source from a small set of noisy interior sensors. That combination is the focus of the present work, and the sparse-sensor setting is precisely where physics-informed constraints matter most: sixteen sensors cannot determine a 192×192 field from data alone, so the governing equation and the boundary and initial conditions must carry the spatial information that the measurements do not.

Table 1: Positioning of the present work with respect to recent physics-informed neural-network heat-conduction studies.

Study	Task and setting	Estimated unknowns	Sensors / data	Reported accuracy
Cai <i>et al.</i> [4]	Forward/inverse; 2-D convection, cylinder	Temperature, velocity, and pressure fields; boundary conditions; Nusselt number	As few as six sparse sensors	Relative L_2 temperature errors, 1.25–15.09%
Zobeiry and Humfeld [16]	Forward; 1-D and 2-D conduction, convective boundaries	Temperature field only	No sensor data	Agrees with finite-element results
Oommen and Srinivasan [17]	Inverse; four data-sparse heat-transfer cases	Unknowns of the four cases	Sparse measurements	Better than conventional machine learning
He <i>et al.</i> [18]	Forward/inverse; 1-D conduction, wood and steel	Constant material parameters	Measured data	High accuracy against exact solutions
Billah <i>et al.</i> [19]	Inverse; 1-D unsteady aluminum wall	Thermal diffusivity; boundary heat flux	Small interior observation subset	Robust to noisy observations
Go <i>et al.</i> [20]	Forward surrogate; plate of varying thickness	Real-time field surrogate	Transfer learning across scenarios	Agrees with finite-element results
Bidou <i>et al.</i> [21]	Inverse; 2-D transient plate, Gaussian source	Property tensor, or source amplitude, position, and dispersion (separate tasks)	Sensitivity-guided sensor set; noise up to 0.5 K	High accuracy, reported qualitatively
This work	Inverse; 2-D transient; moving plus stationary source; heat-loss term	Diffusivity, heat loss, and source amplitude simultaneously	16 random sensors; 5424 observations; 10^{-3} noise	$E_{L_2} = 2.87 \times 10^{-2}$; parameter errors 5.04, 24.54, and 7.90%

This study makes five contributions. First, it reconstructs a transient two-dimensional field while identifying three positive thermal coefficients from sixteen noisy interior sensors. Second, it compares the physics-informed model with a neural network of identical architecture but without physics-based losses, as well as with noniterative spatiotemporal interpolation. Third, it examines sensor count, temporal sampling, measurement noise, and every loss weight through controlled sensitivity runs. Fourth, it provides a workflow that details the hardware, accuracy, initialization, random seeds, number of parameters, batch sizes, and stopping rule (see the program available at the link at the end of this paper). Fifth, it situates the configuration against the closest published studies, including Bidou *et al.* [21], in Table 1. The analysis treats field reconstruction and parameter observability as distinct objectives and discusses the requirements for transfer to experimental measurements.

2 Physics-informed formulation

Let $x \in [0, L_x]$ and $y \in [0, L_y]$ denote the spatial coordinates, let $t \in [0, T]$ denote time, and let $u(x, y, t)$ be the temperature field. The rectangular spatial domain is $\Omega = (0, L_x) \times (0, L_y)$, its boundary is $\partial\Omega$, and T is the final time. A feedforward network $u_\theta(x, y, t)$ approximates u , where θ collects all trainable neural weights and biases. The unknown positive coefficients are the thermal diffusivity α , the linear heat-loss coefficient γ , and the source-amplitude coefficient β . The strong-form residual is

$$\mathcal{R}_\theta(x, y, t) = \frac{\partial u_\theta}{\partial t} - \alpha \left(\frac{\partial^2 u_\theta}{\partial x^2} + \frac{\partial^2 u_\theta}{\partial y^2} \right) + \gamma u_\theta - \beta S(x, y, t), \quad (1)$$

where $\mathcal{R}_\theta$ measures the local heat-balance error and $S(x, y, t)$ is the prescribed, dimensionless source shape.

Suppose that N_s scalar observations are recorded. The sensor data set is

$$\mathcal{D}_s = \{(x_m, y_m, t_m, \tilde{u}_m)\}_{m=1}^{N_s}, \quad \tilde{u}_m = u(x_m, y_m, t_m) + \varepsilon_m, \quad \varepsilon_m \sim \mathcal{N}(0, \sigma_n^2). \quad (2)$$

Here, m indexes an observation; (x_m, y_m, t_m) is its space-time coordinate; $\tilde{u}_m$ is the measured temperature; ε_m is zero-mean Gaussian noise; σ_n is its standard deviation; and $\mathcal{N}(0, \sigma_n^2)$ denotes a normal distribution with variance σ_n^2 .

Training minimizes the composite objective

$$\mathcal{L} = w_d \mathcal{L}_{\text{data}} + w_f \mathcal{L}_{\text{PDE}} + w_b \mathcal{L}_{\text{BC}} + w_0 \mathcal{L}_{\text{IC}} + w_p \mathcal{L}_{\text{prior}}, \quad (3)$$

where $\mathcal{L}$ is the total loss and w_d , w_f , w_b , w_0 , and w_p are nonnegative weights for the data, governing-equation, boundary-condition, initial-condition, and parameter-prior terms. The abbreviations PDE, BC, and IC denote partial differential equation, boundary condition, and initial condition, respectively. The component losses are

$$\begin{aligned} \mathcal{L}_{\text{data}} &= \frac{1}{N_s} \sum_{m=1}^{N_s} |u_\theta(x_m, y_m, t_m) - \tilde{u}_m|^2, & \mathcal{L}_{\text{PDE}} &= \frac{1}{N_f} \sum_{j=1}^{N_f} |\mathcal{R}_\theta(\xi_j)|^2, \\ \mathcal{L}_{\text{BC}} &= \frac{1}{N_b} \sum_{j=1}^{N_b} |u_\theta(\xi_j^{(b)})|^2, & \mathcal{L}_{\text{IC}} &= \frac{1}{N_0} \sum_{j=1}^{N_0} |u_\theta(x_j^{(0)}, y_j^{(0)}, 0) - u_0(x_j^{(0)}, y_j^{(0)})|^2. \end{aligned} \quad (4)$$

The index j enumerates training points. The points $\xi_j = (x_j^{(f)}, y_j^{(f)}, t_j^{(f)})$ lie inside $\Omega \times (0, T]$, and $\xi_j^{(b)} = (x_j^{(b)}, y_j^{(b)}, t_j^{(b)})$ lie on $\partial\Omega \times (0, T]$. The pair $(x_j^{(0)}, y_j^{(0)})$ is an initial-time spatial point, and $u_0(x, y)$ is the prescribed initial field. The superscripts f , b , and 0 label interior, boundary, and initial sets. Accordingly, N_f , N_b , and N_0 are the numbers of interior, boundary, and initial points in each training batch.

A weak logarithmic prior,

$$\mathcal{L}_{\text{prior}} = (\log \alpha - \log \alpha_0)^2 + (\log \gamma - \log \gamma_0)^2 + (\log \beta - \log \beta_0)^2, \quad (5)$$

conditions the early inverse search, where α_0 , γ_0 , and β_0 are the initial coefficient guesses and $\log$ is the natural logarithm. Positivity is enforced through $\alpha = e^{\eta_\alpha}$, $\gamma = e^{\eta_\gamma}$, and $\beta = e^{\eta_\beta}$; e is the natural exponential base, and η_α , η_γ , and η_β are unconstrained trainable scalars.

The nominal weight vector is $(w_d, w_f, w_b, w_0, w_p) = (50, 1, 10, 10, 10^{-4})$. The data term is scaled upward because its unweighted mean-square magnitude is small relative to the PDE residual. The boundary and initial terms receive larger weights to preserve the known constraints, whereas the prior remains deliberately weak. Section 4.3 perturbs

each weight separately rather than treating the nominal vector as universally optimal. The network has five hidden layers of width 128 and hyperbolic-tangent activations. Its input contains the normalized coordinates (x, y, t) and the sine and cosine of 48 Gaussian random Fourier projections, which help represent the moving localized source. The neural network has 78,977 trainable parameters; the three physical scalars increase the total to 78,980.

3 Thermal benchmark and numerical protocol

The benchmark uses normalized coordinates on the unit square $\Omega = (0, 1)^2$ over the time interval $0 \leq t \leq T = 1$:

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u - \gamma u + \beta S(x, y, t), \quad u|_{\partial\Omega} = 0, \quad u(x, y, 0) = 0. \quad (6)$$

Here, $\nabla^2 u = \partial^2 u / \partial x^2 + \partial^2 u / \partial y^2$ is the two-dimensional Laplacian. The superscript $\star$ identifies the exact coefficient used only to generate the synthetic reference data: $\alpha^\star = 0.017$, $\gamma^\star = 0.85$, and $\beta^\star = 7.5$. The source combines a moving pulsed Gaussian with a weaker stationary hot spot:

$$S(x, y, t) = p(t) \exp \left[-\frac{(x - c_x(t))^2}{2\sigma_x^2} - \frac{(y - c_y(t))^2}{2\sigma_y^2} \right] + A_s p_s(t) \exp \left[-\frac{(x - x_s)^2 + (y - y_s)^2}{2\sigma_s^2} \right], \quad (7)$$

$$c_x(t) = \frac{1}{2} + r_x \cos(2\pi n_r t + \phi), \quad c_y(t) = \frac{1}{2} + r_y \sin(2\pi n_r t + \phi), \quad (8)$$

with $p(t) = 0.65 + 0.35 \sin^2(2\pi n_p t)$ and $p_s(t) = 0.5 + 0.5 \cos^2(2\pi n_s t)$. The pair (c_x, c_y) is the moving-source center; r_x and r_y are its orbit radii; n_r is the number of rotations; and ϕ is the phase. The quantities σ_x and σ_y are the main-source spreads. For the secondary source, A_s is the relative amplitude, (x_s, y_s) is the center, and σ_s is the spread. Finally, n_p and n_s control the two pulse modulations. The numerical values are $r_x = 0.25$, $r_y = 0.18$, $n_r = 1.35$, $\phi = 0.15\pi$, $\sigma_x = 0.055$, $\sigma_y = 0.045$, $A_s = 0.35$, $(x_s, y_s) = (0.27, 0.73)$, $\sigma_s = 0.085$, $n_p = 5$, and $n_s = 2$.

A five-point explicit finite-difference scheme generates the reference field on a 192×192 grid. The temporal grid is automatically increased from 1001 to 2698 points to satisfy the sufficient stability condition

$$\Delta t \leq \frac{2s_f}{\gamma^\star + 4\alpha^\star(\Delta x^{-2} + \Delta y^{-2})}, \quad s_f = 0.92, \quad (9)$$

where Δx , Δy , and Δt are the uniform spatial and temporal steps, and s_f is a dimensionless safety factor. Sixteen interior sensor locations are drawn once with seed 17 and a 12% boundary margin. Every eighth stable time sample is retained, including $t = 1$, which gives 339 samples per sensor and $N_s = 5424$ observations. Gaussian noise with standard deviation $\sigma_n = 10^{-3} \max |u| = 7.47 \times 10^{-4}$ normalized temperature units (n.t.u.) is added with seed 1701.

The training is performed using the MATLAB numerical-computing environment, release R2026a Update 4, Deep Learning Toolbox 26.1, Parallel Computing Toolbox 26.1, single precision, and an NVIDIA GeForce RTX 4050 Laptop graphics processing unit (GPU) with 6 gigabytes (GB) of memory. The adaptive moment estimation (Adam) optimizer runs for a fixed budget of 7200 iterations with an initial learning rate of 2×10^{-3} and decay of 10^{-4} ; early stopping is disabled. Weights are initialized with the Glorot-uniform distribution, with per-layer limit $\sqrt{6/(n_{\text{in}} + n_{\text{out}})}$, where n_{in} and n_{out} are the input and output unit counts, to preserve the scale of activations and gradients across the five tanh layers; biases are zero. The 48 Gaussian random Fourier projections use a fixed scale of 2.5. A fixed seed of 2701 makes the nominal comparison reproducible. The initial coefficient guesses $(\alpha_0, \gamma_0, \beta_0) = (0.008, 0.35, 3.5)$ are deliberately biased away from the true values so that the recovery in Table 2 reflects identifiability rather than a favorable starting point. Each iteration uses all 5424 observations and newly sampled batches of 4096 interior, 1024 boundary, and 1024 initial points. The nominal wall-clock training time is 241.46 s; it is reproducibility metadata only and is never used as a performance statistic.

The data-only neural network uses the same architecture, Fourier map, initialization seed, observations, and 7200-iteration budget, but it minimizes only the sensor mean-square error and does not identify physical parameters. The interpolation baseline first applies linear interpolation in time at each sensor and then natural-neighbor interpolation in space, with nearest-neighbor extrapolation where needed. Both baselines use exactly the same sensor data as the physics-informed model, so differences in field accuracy can be attributed to the physics constraints rather than to data availability.

For evaluation frame k , the root-mean-square error (RMSE) and mean absolute error (MAE) are

$$\text{RMSE}_k = \left(\frac{1}{N_g} \sum_{i=1}^{N_g} (\hat{u}_{i,k} - u_{i,k})^2 \right)^{1/2}, \quad \text{MAE}_k = \frac{1}{N_g} \sum_{i=1}^{N_g} |\hat{u}_{i,k} - u_{i,k}|, \quad (10)$$

where i indexes a grid point, $N_g = 192^2$ is the number of grid points, $\hat{u}_{i,k}$ is the predicted temperature, and $u_{i,k}$ is the reference temperature. Over $N_e = 12$ evaluation frames, the global relative error is

$$E_{L_2} = \left(\frac{\sum_{k=1}^{N_e} \|\hat{\mathbf{u}}_k - \mathbf{u}_k\|_2^2}{\sum_{k=1}^{N_e} \|\mathbf{u}_k\|_2^2} \right)^{1/2}, \quad (11)$$

where $\hat{\mathbf{u}}_k$ and $\mathbf{u}_k$ are the vectorized predicted and reference fields, and $\|\cdot\|_2$ is the Euclidean norm. The zero-energy initial frame is excluded only from framewise relative-error averages; Equation (11) remains well defined over all N_e frames.

4 Results

4.1 Nominal optimization, reconstruction, and parameter identification

Figure 1 summarizes the nominal optimization: the weighted data and PDE terms dominate the final objective, the total loss reaches 1.53×10^{-3} at iteration 7200, and the normalized trajectories show late-stage recovery of α and β , while γ remains below its reference value. Training stops at the fixed iteration budget, so these curves describe optimization behavior rather than an early-stopping criterion.

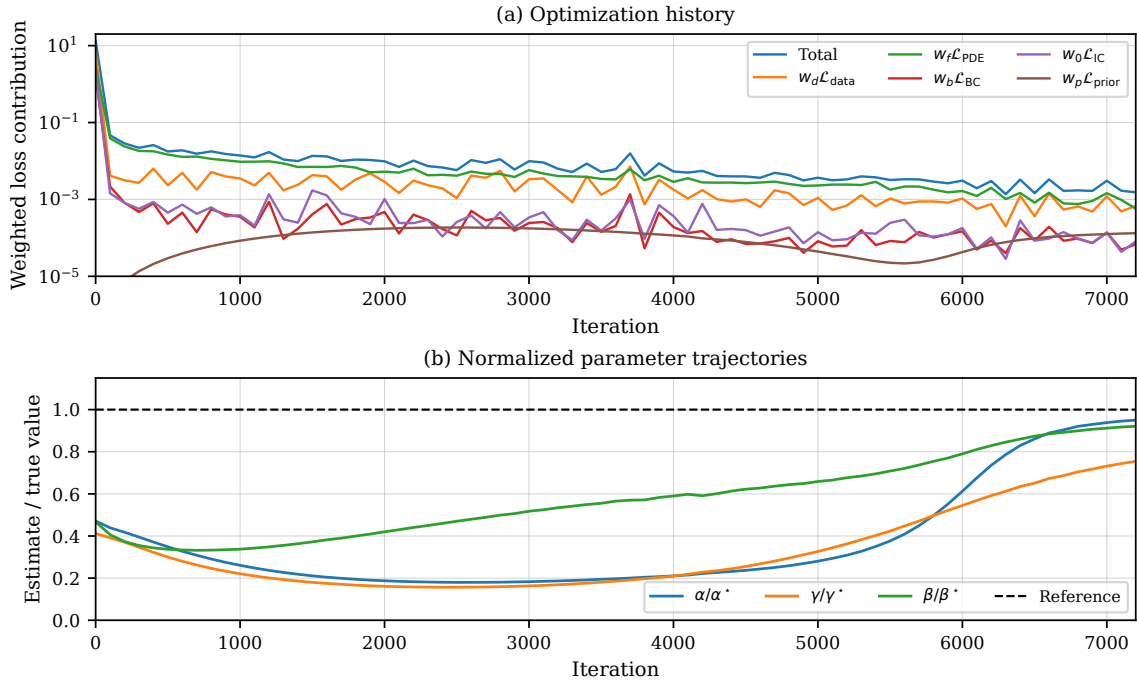


Figure 1: Nominal PINN training. (a) Total loss and weighted components. (b) Identified coefficients normalized by their true values. PDE, BC, and IC denote partial differential equation, boundary condition, and initial condition.

Across the 12 evaluation frames, the mean RMSE is 3.5175×10^{-3} n.t.u., the maximum RMSE is 4.4057×10^{-3} n.t.u., the mean MAE is 2.4203×10^{-3} n.t.u., and $E_{L_2} = 2.8689 \times 10^{-2}$. At $t = 1$, the RMSE is 2.5072×10^{-3} n.t.u. and the relative L_2 error is 1.3563×10^{-2} . As shown in Figure 2, the reconstruction preserves the dominant hot region, the secondary footprint, and the diffusion trail despite measurements from only sixteen locations; the largest discrepancies remain near steep source-induced gradients.

Table 2 shows relative errors below 8% for α and β , but 24.54% for γ . This difference is physically plausible: diffusivity and volumetric loss both affect temporal decay and spatial smoothing and are hard to separate with

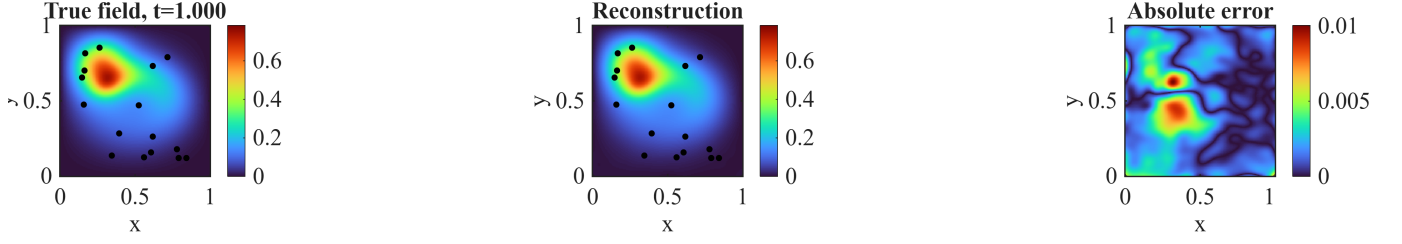


Figure 2: Final-time field at $t = 1$. Left: finite-difference reference; center: PINN reconstruction; right: absolute error. Black points indicate the sixteen sensor locations.

Table 2: Physical-parameter identification in the nominal run.

Parameter	True	Initial	Estimated	Relative error (%)
α	0.0170	0.0080	0.016144	5.036
γ	0.8500	0.3500	0.641444	24.536
β	7.5000	3.5000	6.907141	7.905

sparse sensors, whereas source amplitude is more directly related to peak temperature. Accurate field reconstruction and partial coefficient recovery therefore coexist, and state and parameter errors must be interpreted separately.

4.2 Controlled comparison with data-only reconstruction

Figure 3 and Table 3 provide the main controlled validation. The PINN reduces mean RMSE by 96.03% relative to the matched data-only neural network (NN) and by 95.56% relative to spatiotemporal interpolation. The corresponding reductions in E_{L_2} are 95.98% and 95.77%. At the final frame, the two data-only baselines have relative errors near 0.64, whereas the PINN gives 0.0136. The conventional NN has the same 78,977 neural parameters and the same initialization, so the accuracy gap cannot be explained by network size. Instead, it isolates the regularization provided by the PDE, boundary, and initial-condition terms. Only the PINN also estimates α , γ , and β . We acknowledge that the two iterative models are not symmetric in the information they use: the PINN receives the governing equation, the boundary condition, and the initial condition in addition to the sensor data, whereas the data-only network sees the sensor data alone. This asymmetry is the object of the comparison: it quantifies the value of the known physics for this benchmark. From sixteen scattered sensors a data-only network must extrapolate a 192-by-192 field from severely underdetermined information, and its error concentrates off-sensor (relative errors near 0.64 at the final frame), whereas the physics terms propagate information through the domain and, together with the sensor data, make both the field and the three parameters identifiable.

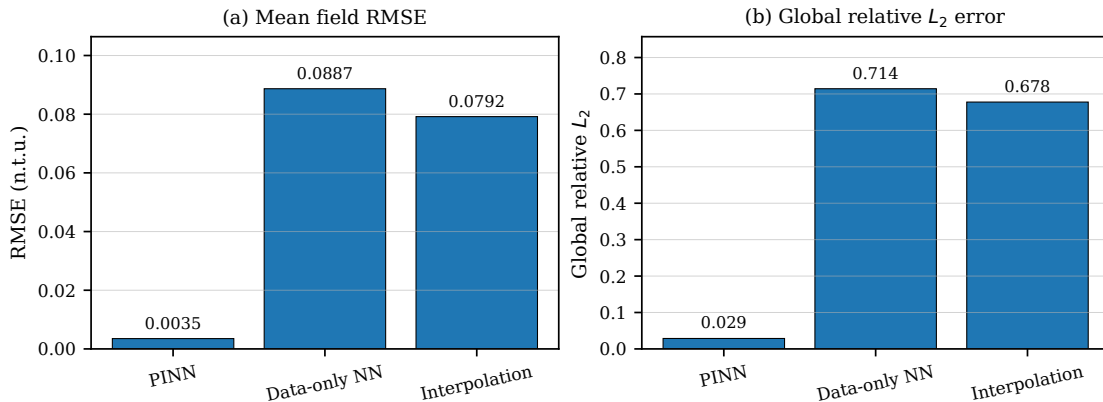


Figure 3: Controlled reconstruction comparison using the same sixteen sensors and 5424 noisy observations. Training time is intentionally excluded from the performance analysis.

This comparison is specific to one benchmark and is not a universal ranking: classical regularized optimization can be highly competitive when the forward model is reliable, and operator-learning surrogates may be preferable when many parameterized queries are available. The supported claim is narrower: for the present sparse moving-source problem, the physics constraints prevent the severe off-sensor degradation observed in the two data-only baselines.

Table 3: Reconstruction comparison on the common benchmark.

Method	Mean RMSE	Mean MAE	Global L_2	Final RMSE	Parameters
PINN	0.003518	0.002420	0.028689	0.002507	78,980
Data-only neural network	0.088679	0.060161	0.714344	0.117861	78,977
Spatiotemporal interpolation	0.079173	0.048275	0.677735	0.119443	0

4.3 Sensitivity and loss-weight analysis

The sensitivity campaign adds 21 PINN trainings: three sensor counts, three temporal strides, four noise levels, and eleven loss-weight configurations; Figure 4 summarizes these controlled perturbations. Every case uses seed 3701 and one run, so each comparison changes only the selected factor. These runs are diagnostic one-factor-at-a-time tests, not estimates of means, variances, or statistical significance, and wall-clock time is excluded.

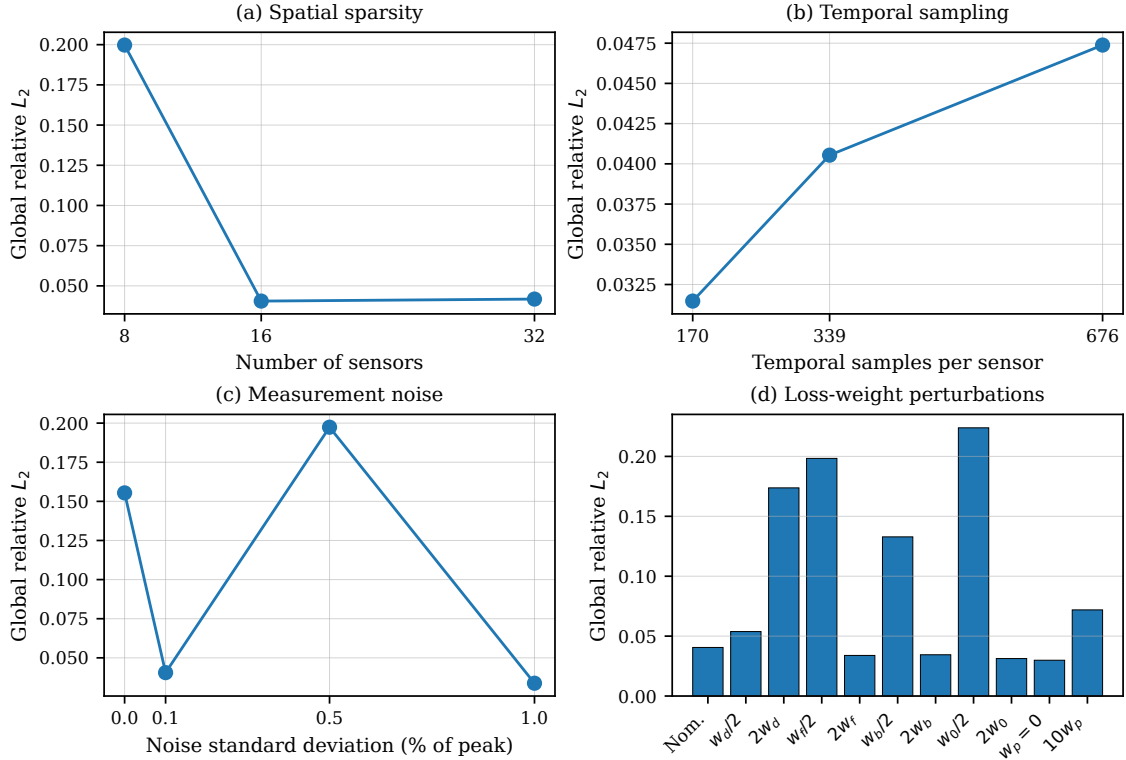


Figure 4: Single-run controlled sensitivity of E_{L_2} . Panels vary (a) sensor count, (b) temporal samples per sensor, (c) noise standard deviation, and (d) one loss weight at a time. “Nom.” denotes the ordered vector $(w_d, w_f, w_b, w_0, w_p) = (50, 1, 10, 10, 10^{-4})$.

Spatial coverage has the clearest effect. With eight sensors, E_{L_2} rises to 0.1998 and the relative errors in (α, γ, β) are (76.03, 70.20, 36.79)%. With sixteen sensors, the corresponding values are 0.0405 and (3.38, 21.84, 6.81)%. Increasing the count to 32 leaves the field error nearly unchanged at 0.0418, but improves the parameter errors to (2.11, 7.26, 2.23)%. Additional spatial coverage therefore improves coefficient observability even after field accuracy has nearly saturated.

Temporal subsampling is less damaging over the tested range. The cases with 170, 339, and 676 samples per sensor give E_{L_2} values of 0.0315, 0.0405, and 0.0474. This nonmonotonic order does not mean that fewer measurements are inherently better. It shows that fixed-budget, nonconvex training can respond to data density through changes in effective loss balance and stochastic collocation. The noise sweep is likewise nonmonotonic: E_{L_2} equals 0.1554, 0.0405, 0.1974, and 0.0338 for standard deviations of 0, 0.1%, 0.5%, and 1% of the peak field. These single-run values are not evidence that noise improves the inverse problem. They instead reveal optimizer variability and motivate repeated seeds, ensembles, or Bayesian uncertainty analysis [13].

The loss perturbations confirm the need for balance. Doubling w_d or halving w_f raises E_{L_2} to 0.1737 and 0.1983 because data fitting dominates without sufficient physics regularization. Doubling w_f , w_b , or w_0 gives errors of 0.0339, 0.0344, and 0.0313. Removing the weak prior gives 0.0299, whereas increasing it tenfold gives 0.0719 and biases all

three coefficients. The prior is therefore an early conditioning device, not a source of final physical information. The nominal vector is a stable reference configuration, but these results favor future adaptive or gradient-balanced weighting over any claim of universal optimality [10].

5 Discussion

The results show that the physics-informed formulation improves temperature-field reconstruction under sparse sensing, while parameter recovery remains more sensitive to observability. The controlled comparison isolates the contribution of the governing-equation and constraint terms, whereas the sensitivity analysis identifies spatial coverage and loss balance as important factors affecting the inverse solution. These findings should be interpreted within the scope of the synthetic benchmark and fixed-budget training protocol.

Table 2 shows relative errors below 8% for α and β , but 24.54% for γ . This difference is physically plausible. Diffusivity and volumetric loss both affect temporal decay and spatial smoothing, making them difficult to separate with sparse sensors; by contrast, source amplitude relates more directly to peak temperature. The result supports accurate field reconstruction and partial coefficient recovery, while confirming that state and parameter errors must be interpreted separately.

This comparison is specific to one benchmark and is not a universal ranking of inverse heat-transfer methods. Classical regularized optimization can be highly competitive when the forward model is reliable and repeated solves are affordable. Operator-learning surrogates may be preferable when many parameterized queries are available. The supported claim is narrower: for the present sparse moving-source problem, the physics constraints prevent the severe off-sensor degradation observed in the two data-only baselines.

5.1 Transfer to experimental measurements and limitations

The validation uses synthetic finite-difference data and therefore does not establish experimental deployment performance. Real thermocouples, resistance sensors, and infrared cameras introduce calibration offsets, gain drift, quantization, finite spatial footprints, temporal lag, missing samples, synchronization error, and sensor-position uncertainty; infrared measurements additionally depend on emissivity, reflected ambient radiation, viewing angle, and background conditions. These effects can exceed the independent additive Gaussian noise assumed here.

The zero initial field of Equation (6) is a property of the synthetic benchmark, not a limitation of the framework: the initial-condition term of the loss accepts any prescribed field $u_0(x, y)$, and an unknown initial state could be estimated jointly or supplied from a separate measurement. Transfer of the reported results to a real thermal system therefore requires a known or measured initial field, a valid two-dimensional model form and boundary representation, and a source model consistent with the actual heat input; under those conditions the identification workflow applies unchanged, while the reported error levels remain specific to this benchmark.

Model discrepancy is equally important: experimental boundaries may exchange heat by convection or radiation rather than satisfy a homogeneous Dirichlet condition, source position and intensity may be uncertain, material properties may vary with temperature, and three-dimensional losses may invalidate a two-dimensional model. A practical workflow should calibrate sensors independently, reserve spatially distinct sensors and time intervals for validation, use robust or uncertainty-weighted data losses, and introduce trainable sensor offsets or gains when justified; temporal and spatial convolution can represent finite sensor response, and uncertain boundary fluxes and source trajectories can be inferred jointly. Ensemble or Bayesian PINNs can provide uncertainty intervals instead of a single deterministic field [13]. Controlled tests on a plate instrumented with traceable contact sensors or a calibrated infrared camera are required before claiming experimental superiority.

The study also has computational and inferential limitations: it uses one nominal run and one run per sensitivity case, so optimizer variability is not quantified statistically; sensor locations are random rather than optimized, the source form is assumed known, and the parameter prior is centered on the initial guesses. The remaining bias in γ indicates structural correlation between diffusion and dissipation. These limitations motivate repeated-seed studies, optimal sensor placement, adaptive loss balancing, uncertainty quantification, hard boundary enforcement, and tests with unknown source trajectories or spatially varying coefficients.

6 Conclusions

A sparse-sensor PINN was evaluated for transient two-dimensional inverse heat conduction with simultaneous recovery of thermal diffusivity, volumetric heat loss, and source amplitude. With sixteen noisy sensors, the nominal

run achieved a mean RMSE of 3.52×10^{-3} n.t.u. and $E_{L_2} = 2.87 \times 10^{-2}$. The parameter errors were 5.04%, 24.54%, and 7.90%, showing that accurate field recovery does not imply equal observability of every coefficient.

The controlled baselines show that governing physics, not network capacity alone, explains reconstruction quality: mean RMSE decreases by more than 95% relative to both the matched data-only neural network and spatiotemporal interpolation. The sensitivity study identifies insufficient spatial coverage and unbalanced loss weights as the most consequential failure modes in the examined configuration. Its nonmonotonic temporal and noise results also confirm that one fixed-budget run is not a statistical robustness assessment. Future work should combine repeated seeds, adaptive loss weighting, optimized sensor placement, uncertainty quantification, and controlled experimental measurements.

Code and data availability. The MATLAB R2026a source code for this work and the result tables used are available at <https://la.mathworks.com/matlabcentral/fileexchange/183734-sparse-sensor-pinn-reconstruction-and-identification>.

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Conflicts of interest. The authors declare no conflicts of interest.

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