

Convergence Analysis of the Kuramoto Model via Mean-Field Coupling for Micro-Generation Networks

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Abstract

This work investigates the convergence and stability of a decentralized synchronization architecture for networks of Permanent Magnet Synchronous Generators (PMSGs) based on mean-field coupling within the Kuramoto framework. The proposed approach replaces explicit pairwise phase interactions with a collective order parameter, enabling each generator to adjust its dynamics relative to the network centroid. Local phase and frequency estimation is performed through a Synchronous Reference Frame Phase-Locked Loop (SRF-dqPLL), while an adaptive-gain control law regulates the synchronization process. The convergence properties are examined through analytical stability considerations and continuous-time simulations of a four-node PMSG network implemented in Simulink. Under heterogeneous initial frequencies, the numerical results demonstrate convergence toward a common operating frequency, followed by a progressive reduction of phase mismatch without sustained oscillations. Furthermore, the mean-field formulation enables a reduction in the computational complexity of the coupling interactions from $O(N^2)$ to $O(N)$, supporting the scalability of the proposed architecture. These findings support the feasibility of the proposed approach for decentralized micro-generation synchronization and provide a basis for future implementation in resource-constrained embedded systems.

Keywords— Kuramoto model, mean-field coupling, Decentralized synchronization

1 Introduction

The synchronization of coupled oscillators has been extensively investigated through the Kuramoto model, which provides a mathematical framework for understanding the emergence of collective behavior in interacting dynamical systems. In particular, Benedetto et al. [1] studied complete phase synchronization in the mean-field limit, establishing theoretical results on the collective evolution of oscillator populations under specific conditions. These developments highlight the relevance of mean-field descriptions for analyzing synchronization phenomena and motivate their application to finite networks of interacting dynamical units. However, extending these theoretical concepts to electromechanical systems requires considering additional factors, including heterogeneous operating conditions, state estimation, and control dynamics.

The increasing integration of distributed energy resources has motivated the development of decentralized control architectures capable of coordinating multiple generation units while reducing their dependence on centralized supervisory systems [2]. In micro-generation networks operating under islanded conditions, maintaining consistent electrical frequency and phase relationships among individual generators is essential for coordinated operation and eventual interconnection. This challenge becomes particularly relevant for Permanent Magnet Synchronous Generators (PMSGs), whose operating frequencies may differ due to variations in mechanical input conditions. Consequently, synchronization strategies must accommodate heterogeneous initial states while providing stable transient responses and remaining suitable for implementation in resource-constrained embedded systems.

The theoretical foundations of oscillator synchronization have been extensively reviewed by Acebrón et al. [3], while Rodrigues et al. [4] examined the Kuramoto model in complex networks, emphasizing the influence of coupling structures on collective dynamics. In conventional pairwise formulations, the synchronization interaction is determined by the phase differences between connected oscillators. For fully connected networks, the direct evaluation of these interactions requires a computational effort that grows quadratically with the number of nodes. Mean-field coupling provides an alternative representation by describing the collective state through a complex

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order parameter that characterizes network coherence. This formulation allows the synchronization interaction to be evaluated relative to a common reference phase, avoiding the explicit calculation of every pairwise phase difference and reducing the associated computational effort.

Beyond the coupling formulation, practical synchronization depends on the availability of reliable phase and frequency estimates. Phase-Locked Loop (PLL) techniques are widely employed in renewable energy systems to extract synchronization variables from electrical measurements. Ali et al. [5] reviewed three-phase PLL algorithms and examined their performance under different operating conditions, highlighting the importance of estimation accuracy and transient response. In this context, the Synchronous Reference Frame PLL (SRF-dqPLL) provides a suitable mechanism for estimating local electrical states without requiring direct mechanical position measurements. Combining such estimators with distributed synchronization algorithms offers a pathway toward sensorless coordination of micro-generation units.

Recent work by Rincón-Silva et al. [6] explored a low-cost embedded architecture for PMSG micro-generator synchronization based on the Kuramoto model, demonstrating the relevance of oscillator-based coordination strategies for practical micro-generation applications. Building on this application context, the present study focuses on the convergence and stability properties of a mean-field synchronization architecture incorporating local state estimation and adaptive coupling. Particular attention is given to the relationship between frequency convergence, phase cohesion, and transient behavior, as well as to the computational implications of replacing explicit pairwise interactions with a collective order parameter.

Accordingly, this work presents a framework for decentralized synchronization in PMSG micro-generator networks. The proposed architecture combines SRF-dqPLL state estimation, mean-field Kuramoto coupling, and an adaptive-gain control law. Its convergence properties are examined through closed-loop stability analysis and continuous-time simulations of a four-node digital twin under heterogeneous initial frequencies. The main contribution is the integrated assessment of synchronization dynamics, phase-cohesive consensus, and computational scalability within a common control framework. The remainder of the paper presents the mathematical formulation and control methodology in Section 2, the numerical results in Section 3, their interpretation and comparison with previous studies in Section 4, and the main conclusions in Section 5.

2 Methodology

2.1 Analytical Dynamics of the Network Nodes

The proposed micro-generation network consists of N interconnected nodes, each representing an individual Permanent Magnet Synchronous Generator (PMSG) equipped with a local phase and frequency estimation unit and an adaptive synchronization controller. In this study, a four-node configuration ($N = 4$) is considered, where each generator operates under different initial frequency conditions. As illustrated in Fig.1, the SRF-dqPLL estimates the electrical phase and frequency of each generator from local voltage measurements, while the estimated states are exchanged through a shared communication medium. This information allows each node to independently compute the mean-field reference and adjust its dynamics through the adaptive control law, without requiring a centralized controller.

The proposed architecture employs a reduced-order analytical model for the electromechanical dynamics of a PMSG. Assuming a uniform air gap and neglecting magnetic saturation, Faraday's law dictates that the induced voltages (v_a, v_b, v_c) at the stator terminals are equal to the time variation of the stator magnetic flux linkage. Consequently, the induced voltage amplitude is directly proportional to the permanent-magnet flux linkage amplitude λ_m and to the electrical angular velocity $\omega_i(t)$ [5, 7]. The continuous-time voltage dynamics for phase a are defined by Eq. (1):

$$v_{a,i}(t) = \omega_i(t)\lambda_m \sin(\theta_i(t)) \quad (1)$$

where $\theta_i(t) = \int_0^t \omega_i(\tau) d\tau$ represents the instantaneous electrical angular position. To ensure dimensional consistency in Eq. (1), where $\omega_i(t)$ is expressed in rad/s and $v_{a,i}(t)$ in V, a permanent-magnet flux linkage constant of $\lambda_m = 0.0876$ Wb is established for the digital-twin parameterization, which proportionally determines the amplitude of the generated voltages as a function of the angular velocity of each node.

Figure 2 presents the electromechanical and single-line representation of the i -th PMSG unit (Node i), defining the physical and electrical framework of the multi-generator network. The mechanical subsystem represents the rotor dynamics described by Eq.14, where the prime mover supplies the mechanical torque $T_m(t)$, counteracted by the electromagnetic torque $T_e(t)$ acting on a shaft characterized by inertia J , damping D , speed $\omega_m(t)$, and position

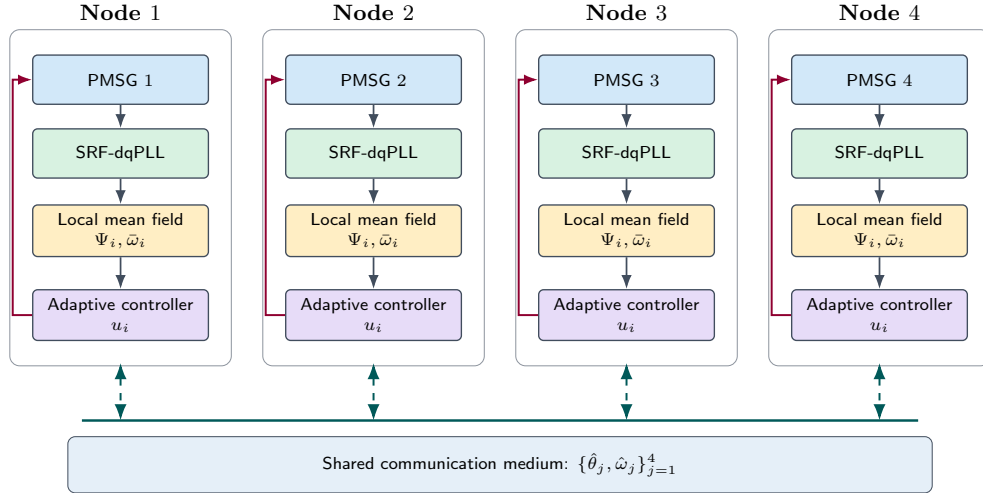


Figure 1: General architecture of the proposed four-node PMSG micro-generation network, illustrating local state estimation, adaptive synchronization control, and information exchange for decentralized mean-field coupling.

$\theta_m(t)$. On the electrical side, the PMSG is represented by a three-phase single-line model, connected in series with the stator resistance R_s and inductance L_s . The terminal voltages $v_{abc,i}(t)$, measured before the grid-connection switch, are fed to the Local Controller & State Estimator. Within this block, the SRF-dqPLL and adaptive controller estimate the electrical frequency $\hat{\omega}_i(t)$ and phase angle $\hat{\theta}_i(t)$.

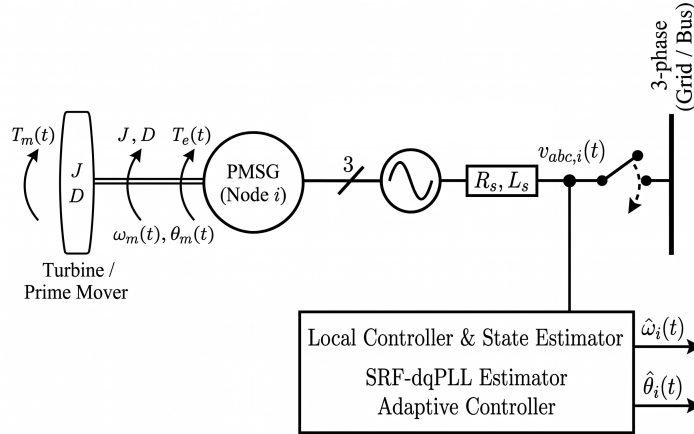


Figure 2: Electromechanical circuit model of the i -th PMSG node connected to the grid, including the local controller and state estimator.

2.2 State Estimation (SRF-dqPLL) in Continuous Time

To operate under a decentralized scheme without mechanical sensors, each node must estimate its instantaneous frequency and phase exclusively from voltage measurements. Throughout this manuscript, the accent ($\hat{\cdot}$) is employed to denote state variables that are estimated or reconstructed by the control algorithm, thereby distinguishing them from the actual physical states of the system.

For the network to operate under a decentralized scheme and, in practice, without mechanical sensors, each node must estimate its instantaneous frequency and phase exclusively from voltage measurements. The implemented algorithm is a Synchronous Reference Frame Phase-Locked Loop (SRF-dqPLL) [6]. The continuous three-phase signals (v_a, v_b, v_c) are first projected onto a stationary orthogonal ($\alpha\beta$) frame through the Clarke transformation, incorporating a scaling factor of $2/3$ to guarantee amplitude invariance, and are subsequently projected onto a rotating (dq) frame through the Park transformation [5], as expressed in Eq. (2):

$$\begin{bmatrix} v_d(t) \\ v_q(t) \end{bmatrix} = \begin{bmatrix} \cos(\hat{\theta}_i(t)) & \sin(\hat{\theta}_i(t)) \\ -\sin(\hat{\theta}_i(t)) & \cos(\hat{\theta}_i(t)) \end{bmatrix} \begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} \quad (2)$$

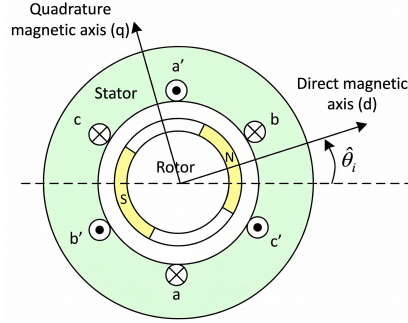


Figure 3: Cross-section of the Permanent Magnet Synchronous Machine (PMSG) illustrating the direct (d) and quadrature (q) rotating reference frames [8].

As illustrated in the phasor diagram of Fig.3, the d (direct) and q (quadrature) axes constitute a rotating reference frame synchronous with the estimated electrical position $\hat{\theta}_i(t)$. The objective of the control loop is to align the system's voltage vector strictly with the direct axis (d -axis). By doing so, the feedback loop forces the orthogonal quadrature component, $v_q(t)$, to approach zero. For small angular deviations between the actual physical phase and the estimated phase, the linearization $v_q(t) \approx V_m(\theta_i(t) - \hat{\theta}_i(t))$ is assumed, where V_m denotes the nominal peak amplitude of the phase voltage. The estimated electrical frequency $\hat{\omega}_i(t)$ is obtained by processing the error $v_q(t)$ through a continuous Proportional-Integral (PI) controller, as expressed in Eq. (3):

$$\hat{\omega}_i(t) = \omega_{nom} + K_p v_q(t) + K_i \int_0^t v_q(\tau) d\tau \quad (3)$$

where ω_{nom} represents the nominal angular frequency of the network, and K_p and K_i are the proportional and integral gains of the tracking loop, respectively. Finally, a virtual Voltage-Controlled Oscillator (VCO) integrates this frequency to obtain the instantaneous estimated phase $\hat{\theta}_i(t)$, thereby closing the feedback loop.

The signal flow described above, from the projection of the measured three-phase voltages to phase estimation through the PI controller and the VCO, is summarized in Fig. 4. This scheme illustrates the feedback of the estimated phase $\hat{\theta}_i(t)$ to the Park transformation, which is essential to guarantee that the error $v_q(t)$ approaches zero.

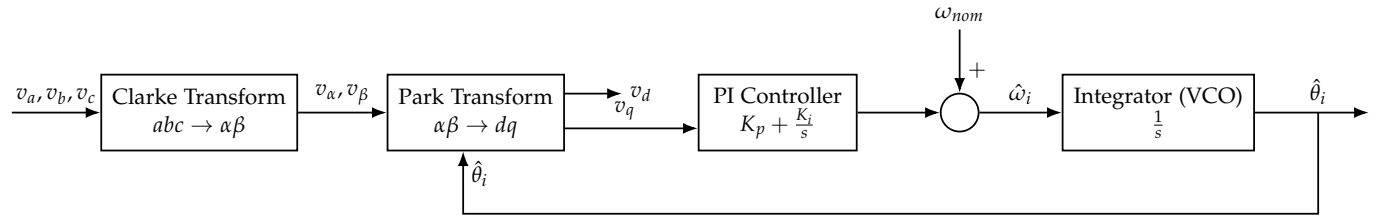


Figure 4: Block diagram of the Synchronous Reference Frame Phase-Locked Loop (SRF-dqPLL).

2.3 Mean-Field Coupling

In the classical Kuramoto model, the computational effort required to synchronize the network grows quadratically because it depends on the sum of pairwise angular differences $\sum \sin(\theta_j - \theta_i)$. To mitigate this algorithmic burden, the model is reformulated using mean-field coupling [9].

The system summarizes the network state through a complex order parameter $Z(t)$, given in Eq. (4), which represents the centroid of a set of M active micro-generators [1], :

$$Z(t) = R(t)e^{i\Psi(t)} = \frac{1}{M} \sum_{j=1}^M e^{i\hat{\theta}_j(t)} \quad (4)$$

The parameter $R(t)$ denotes the network coherence ($0 \leq R \leq 1$), serving as a metric of convergence. The phase of the network centroid, $\Psi(t)$, is defined by Eq. (5):

$$\Psi(t) = \text{atan2} \left(\sum_{j=1}^M \sin(\hat{\theta}_j(t)), \sum_{j=1}^M \cos(\hat{\theta}_j(t)) \right) \quad (5)$$

In Eq. (5), although the atan2 function restricts the centroid phase $\Psi(t)$ to the principal interval $[-\pi, \pi]$ rad, the locally estimated phases $\hat{\theta}_j(t)$ can grow continuously beyond these bounds as time progresses. Because these local angles are evaluated through periodic sine and cosine functions, any phase value falling outside $[-\pi, \pi]$ is naturally mapped without loss of information. Furthermore, computing an unwrapped phase is unnecessary because the subsequent synchronization control law also evaluates the phase difference through a sinusoidal function. Owing to the 2π -periodicity of the sine operator, the periodic resets of $\Psi(t)$ at the $\pm\pi$ boundaries do not introduce discontinuities into the control signal, and operating directly with bounded phases prevents numerical overflow during long-term operation.

Through this transformation, the individual control error $e_{\theta,i}(t)$ of each node is decoupled from individual pairwise interactions and is defined solely by its distance from the unified centroid given in Eq. (6):

$$e_{\theta,i}(t) = \Psi(t) - \hat{\theta}_i(t) \quad (6)$$

This topological abstraction is illustrated in Fig. 5. Panel (a) represents the pairwise model, in which computing the synchronization forces requires evaluating each individual link, resulting in quadratic growth $\mathcal{O}(N^2)$. In contrast, panel (b) demonstrates the efficiency of the mean-field architecture: by directing the attraction toward the global parameter Ψ , the interaction becomes linear, $\mathcal{O}(N)$, eliminating bottlenecks and freeing computational resources for continuous control of the micro-generators.

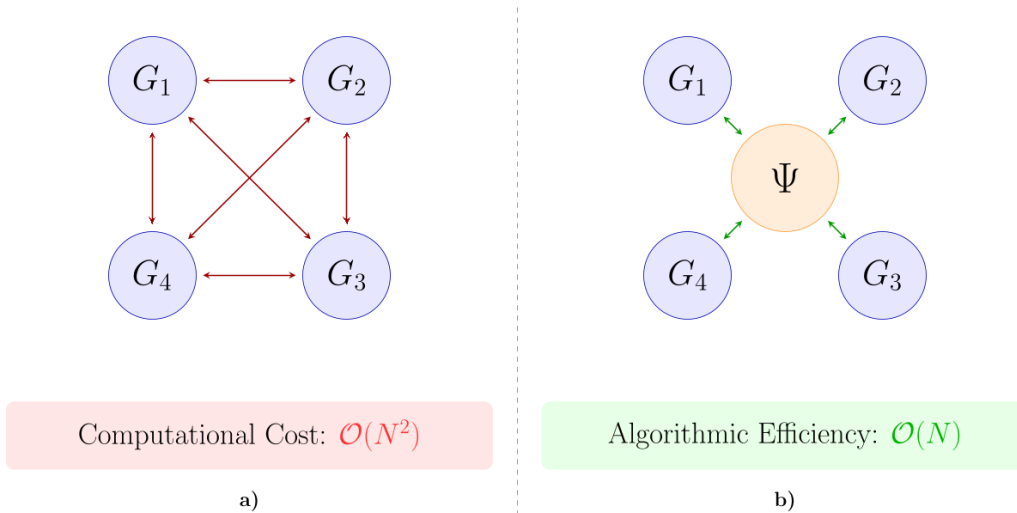


Figure 5: Algorithmic comparison: (a) pairwise interaction, $\mathcal{O}(N^2)$, versus (b) the proposed mean-field coupling, $\mathcal{O}(N)$.

To clarify the operational efficiency and scalability of the proposed decentralized mean-field architecture, it is necessary to distinguish between communication and computational complexities. To avoid relying on a centralized master controller, the architecture assumes a shared local communication medium. Under this scheme, each active node independently transmits its locally estimated phase $\hat{\theta}_i(t)$ exclusively once per synchronization cycle, yielding a linear network-wide communication overhead of $\mathcal{O}(N)$ messages. From a computational perspective, each local node reads the common bus and independently aggregates the M active states to calculate the centroid $\Psi(t)$ using Eq. (5). Consequently, the individual computational burden associated with this arithmetic aggregation scales as $\mathcal{O}(N)$ additions per node. Since the N nodes each perform $\mathcal{O}(N)$ arithmetic additions to estimate the centroid, the total arithmetic complexity across the network scales as $\mathcal{O}(N^2)$. However, the primary advantage of the proposed mean-field formulation lies in the execution of transcendental non-linear operations. Unlike classical pairwise Kuramoto formulations, which require the explicit evaluation of $N(N-1)$ non-linear sine functions and therefore exhibit a global non-linear computational bottleneck of $\mathcal{O}(N^2)$, the mean-field approach evaluates trigonometric interactions linearly. Specifically, each node computes only a single trigonometric deviation with respect to the centroid, $\sin(\Psi - \hat{\theta}_i)$, within the control law. As a result, the total network complexity associated with computationally expensive non-linear operations is reduced to $\mathcal{O}(N)$. This distinction minimizes processing bottlenecks and makes the architecture highly advantageous for resource-constrained embedded applications.

2.4 Adaptive Control Law and Phase-Cohesive Consensus

The synchronization dynamics require the micro-generators to adjust their frequencies in order to reach network consensus. Unlike classical formulations that employ a constant coupling strength, this architecture proposes a control law with an adaptive gain $K_{KUR,i}(t)$ driven by the frequency error relative to the mean field.

First, the instantaneous average of the estimated network frequencies, $\bar{\omega}(t)$, is calculated. The frequency error of each node relative to this average is processed through a saturated Proportional-Integral (PI) controller, defining the dynamic coupling strength, as defined in Eq. (7):

$$K_{KUR,i}(t) = \text{sat} \left(K_p |\bar{\omega}(t) - \hat{\omega}_i(t)| + K_i \int |\bar{\omega}(\tau) - \hat{\omega}_i(\tau)| d\tau \right) \quad (7)$$

This dynamic gain multiplies the nonlinear phase error, generating the final correction signal, as expressed in Eq. (8):

$$u_i(t) = K_{KUR,i}(t) \sin(\Psi(t) - \hat{\theta}_i(t)) \quad (8)$$

The implementation of this adaptive scheme guarantees smooth asymptotic convergence. Because it depends on the integral of the frequency error, the gain acts as a soft-start mechanism.

To show that convergence of the local phase error implies consensus ($R(t) \rightarrow 1$), the local phase is written as $\hat{\theta}_i(t) = \Psi(t) - e_{\theta,i}(t)$ and substituted into the definition of the order parameter, yielding Eq. (9):

$$R(t)e^{i\Psi(t)} = e^{i\Psi(t)} \left(\frac{1}{M} \sum_{j=1}^M e^{-ie_{\theta,j}(t)} \right) \quad (9)$$

Factoring out the term $e^{i\Psi(t)}$ isolates the scalar magnitude $R(t)$, which represents network coherence given in Eq. (10):

$$R(t) = \left(\frac{1}{M} \sum_{j=1}^M e^{-ie_{\theta,j}(t)} \right) \quad (10)$$

Applying Euler's identity to expand the complex exponential, the equation can be decomposed into its real and imaginary components using Eq. (11):

$$R(t) = \frac{1}{M} \sum_{j=1}^M \cos(e_{\theta,j}(t)) - i \frac{1}{M} \sum_{j=1}^M \sin(e_{\theta,j}(t)) \quad (11)$$

The imaginary component vanishes because $R(t)$ is defined as a real scalar magnitude. Therefore, the magnitude of the network coherence reduces to the sum of its real components, as shown in Eq. (12):

$$R(t) = \frac{1}{M} \sum_{j=1}^M \cos(e_{\theta,j}(t)) \quad (12)$$

This equation analytically demonstrates the main objective of the adaptive-gain control law. When the tracking algorithm acts and the system reaches steady state, the phase error at each individual node is driven toward zero ($e_{\theta,i}(t) \rightarrow 0$).

Under this condition, the trigonometric limit satisfies $\lim_{e_{\theta,j} \rightarrow 0} \cos(e_{\theta,j}(t)) = 1$. Substituting this value yields the final state of the system observed in Eq. (13):

$$R(t) \rightarrow \frac{1}{M} \sum_{j=1}^M (1) = \frac{M}{M} = 1 \quad (13)$$

This derivation shows that eliminating the local angular deviations through the controller is a sufficient condition for achieving electromagnetic coherence of the network and, consequently, a state of phase consensus.

While the proposed adaptive gain control law drives the phase error to zero ($e_{\theta,i}(t) \rightarrow 0$), it is necessary to clarify the range of attraction for this consensus. Due to the 2π periodicity of the sinusoidal coupling function (Eq. (8)), the dynamical system evolves in such a way that the phases of the oscillators can be separated by exactly π radians. Consequently, the synchronization achieved is not strictly global for all arbitrary initial conditions over $(-\pi, \pi]$. Instead, the consensus is formally restricted to a cohesive phase region, defined by a maximum initial phase spread strictly less than π radians ($\max |\hat{\theta}_i(0) - \hat{\theta}_j(0)| < \pi$).

2.5 Rotor-Dynamics Modeling and Closed-Loop Stability

The rotational dynamics of each PMSG rotor are governed by Newton's second law for rotation, which balances the inertial acceleration and viscous rotational damping against the net torque acting on the shaft [10]; as expressed in Eq. (14)

$$J \frac{d^2 \theta_m(t)}{dt^2} + D \frac{d \theta_m(t)}{dt} = T_m(t) - T_e(t) \quad (14)$$

where J is the total moment of inertia of the generator rotor, D denotes the viscous friction, $\theta_m(t)$ and $\frac{d \theta_m(t)}{dt}$ represent the mechanical angular position and mechanical angular velocity, respectively, $T_m(t)$ is the mechanical driving torque, and $T_e(t)$ is the opposing electromagnetic torque.

Applying the Laplace transform, we obtain the Eq. (15):

$$(Js + D)\Omega(s) = T_m(s) - T_e(s) \quad (15)$$

and therefore the transfer function between the torque difference and angular velocity is given in Eq. (16):

$$\frac{\Omega(s)}{T_m(s) - T_e(s)} = \frac{1}{Js + D} \quad (16)$$

This result shows how the inclusion of the damping term D transforms the original differential equation into a more stable first-order model consistent with the dynamics observed in real electrical systems.

If the synchronization dynamics of a generator are modeled as a first-order system with inertia (J) and natural damping (D), its transfer function $G(s)$ is given by Eq. 16.

The Proportional-Integral (PI) controller in the Laplace domain [11] as expressed in Eq. (17):

$$C(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s} \quad (17)$$

The closed-loop stability condition is obtained by setting the denominator of the overall transfer function equal to zero [11], using Eq. (18):

$$1 + C(s)G(s) = 0 \quad (18)$$

Substituting $C(s)$ and $G(s)$ gives the Eq. (19):

$$1 + \left(\frac{K_p s + K_i}{s} \right) \left(\frac{1}{Js + D} \right) = 0 \quad (19)$$

Multiplying by $s(Js + D)$ to eliminate the denominators and collecting terms yields Eq. (20):

$$Js^2 + (D + K_p)s + K_i = 0 \quad (20)$$

This quadratic polynomial constitutes the characteristic equation of the closed-loop system and shows how the PI controller modifies the coefficients associated with inertia, damping, and integral gain.

The Proportional-Integral controller for the adaptive gain was tuned through an analysis of the transient behavior. To validate the closed-loop stability of the system, the characteristic equation is analyzed using the Routh-Hurwitz stability criterion [11]. By constructing the Routh array for the polynomial $Js^2 + (D + K_p)s + K_i = 0$, the terms in the first column result in J for the s^2 row, $(D + K_p)$ for the s^1 row, and K_i for the s^0 row. The criterion establishes that stability is guaranteed if there are no sign changes in this first column. Given that the physical parameters of rotor inertia (J) and damping (D) are intrinsically positive constants, the controller design with strictly positive PI gains ($K_p > 0, K_i > 0$) ensures that $J > 0$, $D + K_p > 0$, and $K_i > 0$. Consequently, with no sign changes present, no root of the characteristic equation lies in the right half of the complex s -plane, confirming the stability of the system.

Furthermore, to theoretically validate the transient response dynamics, the denominator of the closed-loop transfer function is equated to the standard normalized second-order form: $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ [11]. By dividing the characteristic equation (Eq. (20)) by J , the natural frequency of the system is analytically defined as $\omega_n = \sqrt{K_i/J}$, and the damping ratio as $\zeta = (D + K_p)/(2\sqrt{JK_i})$. As demonstrated by the unit-step response curves for second-order systems, damping ratios less than unity ($\zeta < 1$) induce an underdamped behavior characterized by overshoots and oscillations [11]. Therefore, to prevent destructive circulating currents caused by such oscillations in the stators, it is imperative to impose a critically damped ($\zeta = 1.0$) or overdamped response. To achieve this, the proportional gain

must satisfy the theoretical boundary condition: $K_p \geq 2\sqrt{K_i} - D$. The selected parameters ($K_p = 0.035, K_i = 0.02$) were tuned to intrinsically satisfy this analytical constraint against the nominal parameters of the micro-generators, thereby guaranteeing the smooth, oscillation-free exponential convergence observed in the simulation results.

Likewise, the output of this controller is bounded by a saturation block that acts as a physical limiter, preventing excessive control effort during the maximum divergence of the initial transient regime.

To guarantee the nonlinear stability of the closed-loop mean-field synchronization, a Lyapunov-based analysis is applied. Following literature on phase-coupled oscillator networks and the Kuramoto potential [12], we define a continuous, positive-definite Lyapunov candidate function $V(\theta)$ representing the cohesion energy of the network's phase errors. Let $e_{\theta,i} = \Psi - \hat{\theta}_i$ be the local phase error. The potential function is defined by Eq. (21):

$$V = \sum_{i=1}^M (1 - \cos(e_{\theta,i})) \quad (21)$$

This function satisfies $V > 0$ for $e_{\theta,i} \in (-\pi, \pi)$ excluding the origin, and equals zero at the consensus state ($e_{\theta,i} = 0$). Taking the time derivative yields the exact gradient-descent dynamics of the network, as expressed in Eq. (22):

$$\dot{V} = \sum_{i=1}^M \sin(e_{\theta,i}) \dot{e}_{\theta,i} \quad (22)$$

To evaluate Eq. (22), the system is analyzed within a synchronous reference frame rotating at the centroid's velocity ($\omega_{sync} = \dot{\Psi}$). In this frame, the centroid is stationary ($\dot{\Psi}_{sync} = 0$), simplifying the error dynamics to $\dot{e}_{\theta,i} = -\dot{\hat{\theta}}_{i,sync}$. Furthermore, assuming a timescale separation where the inner mechanical dynamics are heavily damped, the phase kinematics are directly governed by the control effort from Eq. (8), yielding $\dot{\hat{\theta}}_{i,sync} \approx u_i(t) = K_{KUR,i}(t) \sin(e_{\theta,i})$. Substituting the resulting relationship, $\dot{e}_{\theta,i} = -K_{KUR,i}(t) \sin(e_{\theta,i})$, into Eq. (22) directly yields Eq. (23):

$$\dot{V} = - \sum_{i=1}^M K_{KUR,i}(t) \sin^2(e_{\theta,i}) \quad (23)$$

Since the adaptive PI controller guarantees positive dynamic gain ($K_{KUR,i}(t) > 0$), it follows from Eq. (23) that $\dot{V} \leq 0$. While this establishes Lyapunov stability, it does not strictly guarantee asymptotic convergence because $\dot{V}$ is negative semi-definite. To prove asymptotic convergence, LaSalle's Invariance Principle is applied [13]. According to this principle, the system trajectories converge to the largest invariant set contained within the region where $\dot{V} = 0$. From Eq. (23), the condition $\dot{V} = 0$ requires $\sin(e_{\theta,i}) = 0$ for all nodes. Because the network operates within the phase-cohesive region $e_{\theta,i} \in (-\pi, \pi)$, the unique valid solution within this domain is the consensus origin $e_{\theta,i} = 0$. Thus, LaSalle's principle formally guarantees that the micro-generators asymptotically converge to the synchronized state. To numerically corroborate this theoretical stability. As illustrated in Fig. 6, prior to synchronization ($t < 1$ s), the network exhibits high, oscillatory potential energy due to decoupled frequencies. Upon activation of the mean-field control law, the energy dissipates ($\dot{V} \leq 0$) and converges to zero. This real-time transient behavior validates the analytical proof, confirming that the adaptive gain strategy drives the multi-generator system toward a stable, cohesive phase consensus.

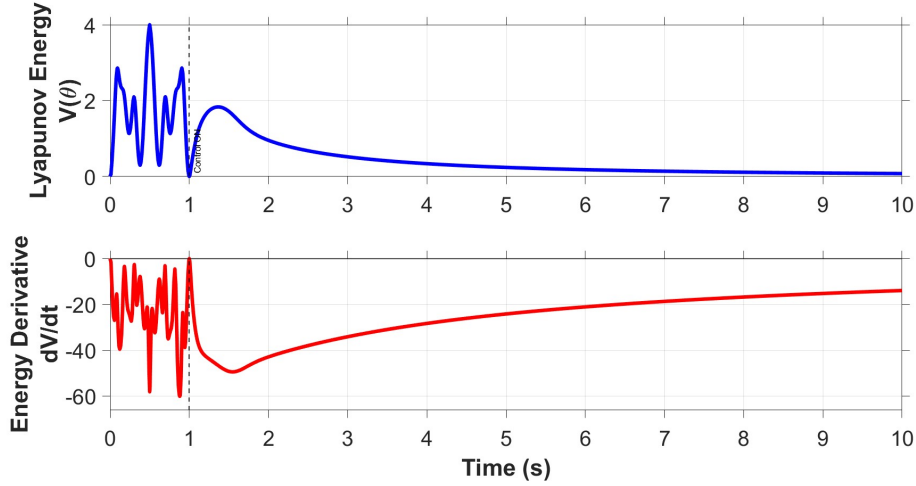


Figure 6: Lyapunov stability analysis. The top panel displays the evolution of the potential energy $V(\theta)$ converging to zero, while the bottom panel confirms that the energy derivative $\dot{V}$ remains negative ($\dot{V} \leq 0$) after the control activation at $t = 1$ s.

2.6 Digital-Twin Configuration and Simulation Conditions

To evaluate the performance of the proposed architecture, the network was implemented in the Simulink environment using a topology composed of $N = 4$ PMSG micro-generator nodes. To test system robustness under initial mismatches, heterogeneous natural oscillation frequencies were assigned to the generators: $f_1 = 17$ Hz, $f_2 = 25$ Hz, $f_3 = 20$ Hz, and $f_4 = 22$ Hz.

The simulation is divided into two temporal stages using a step function. During the first second ($0 \leq t < 1$ s), the network operates in open loop ($K_{KUR,i} = 0$), allowing each SRF-dqPLL algorithm to correctly estimate the natural phase and frequency of its respective node from voltage measurements. At $t = 1$ s, the mean-field control law is activated, closing the synchronization loop.

3 Results

To validate the proposed control architecture and evaluate the stability of mean-field consensus, a continuous digital twin of the system was developed on the Simulink platform. The simulations analyze the transient and steady-state response of a micro-generator network under initial frequency mismatches.

3.1 Validation of the State Estimator (SRF-dqPLL)

A fundamental prerequisite for the mean-field scheme is the correct extraction of the local phase and frequency at each node. During the open-loop interval, the ability of the continuous tracking loop to drive the quadrature error $v_q(t)$ to zero and lock onto the natural dynamics of each micro-generator was evaluated.

Fig.7 illustrates the standalone performance of the decentralized SRF-dqPLL estimator for each of the four nodes during the asynchronous startup phase. Two key behaviors validate the mathematical modeling. First, the peak amplitudes of the measured three-phase terminal voltages (solid lines) vary proportionally with their respective initial mechanical frequencies which strictly complies with the physical relationship established by the permanent-magnet flux linkage λ_m in Eq. (1). Second, the control effectiveness is visually confirmed by the dashed green line, which represents a normalized sinusoidal projection of the estimated phase $\hat{\theta}_i(t)$. Across all four subplots, the zero-crossings and peaks of this estimated signal perfectly align with the phase- a voltage (V_a). This superimposition demonstrates that the controller forces the quadrature component $v_q(t)$ to zero, achieving d -axis alignment for each node. The absence of a steady-state phase offset confirms that the SRF-dqPLL algorithm provides a stable angular reference $\hat{\theta}_i(t)$, which is required to compute the centroid $\Psi(t)$.

3.2 Frequency Convergence Analysis

Once the local measurements of each generator had been validated, the network response to activation of the coupling was evaluated. Fig. 8 presents the time evolution of the electrical frequencies of the four micro-generators.

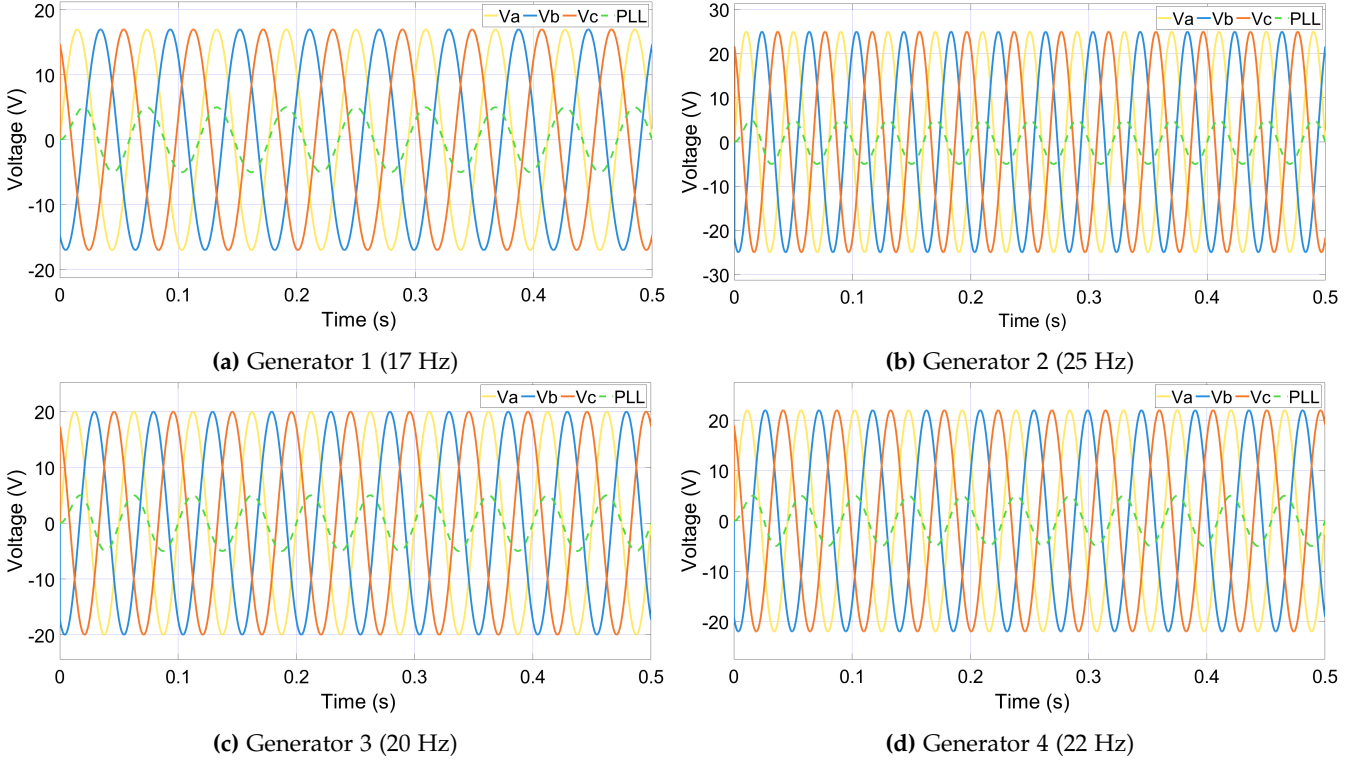


Figure 7: Response of the SRF-dqPLL tracking loop during startup of the four nodes.

Before $t = 1$ s, the system exhibits a dispersed state due to the different natural angular velocities (ω_m). Once the loop is closed and the mean-field coupling is activated, the frequencies converge cooperatively toward a consensus value of approximately 21 Hz, demonstrating that the adaptive gain $K_{KUR,i}(t)$ mitigates the initial frequency divergence.

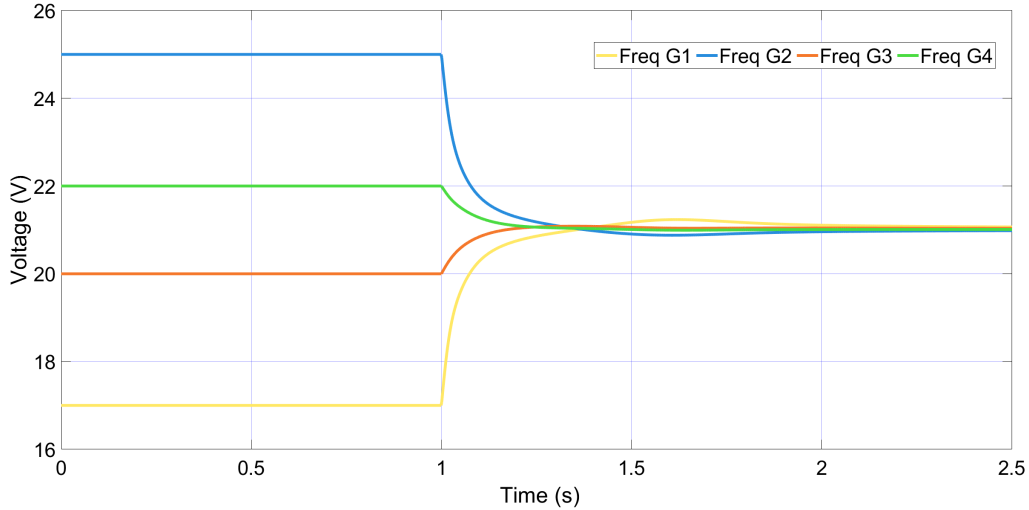


Figure 8: Convergence of the frequencies toward consensus through mean-field coupling.

3.3 Phase Synchronization and Network Consensus

In addition to frequency synchronization, phase consensus is necessary to enable electrical interconnection. Fig.9 shows the decay of the global phase error. After activation at $t = 1$ s, the error exhibits exponential decay without underdamped oscillations. By $t = 8$ s, the error is practically zero, ensuring that the coherence parameter reaches its maximum value.

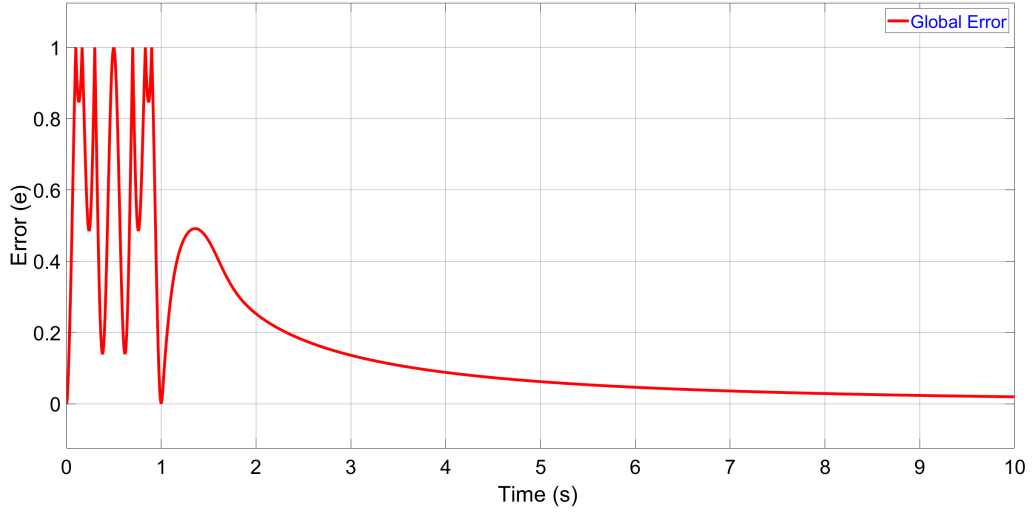


Figure 9: Time evolution of the global phase error.

The electromagnetic impact of this mathematical consensus becomes evident when the voltages of the four network nodes are compared. Fig. 10 shows the superimposed three-phase voltages before synchronization, illustrating an out-of-phase condition that would prevent interconnection to a common bus.

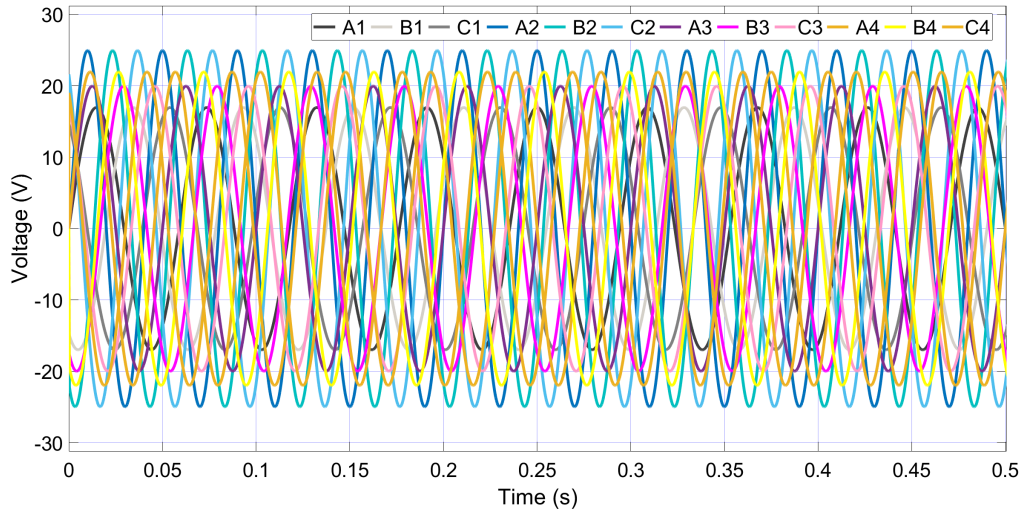


Figure 10: Network voltages in the dispersed state before synchronization.

In contrast, Fig.11 presents the system state after steady state has been reached. The mean-field centroid $\Psi(t)$ aligns the sinusoidal waveforms, resulting in a synchronized three-phase electrical system.

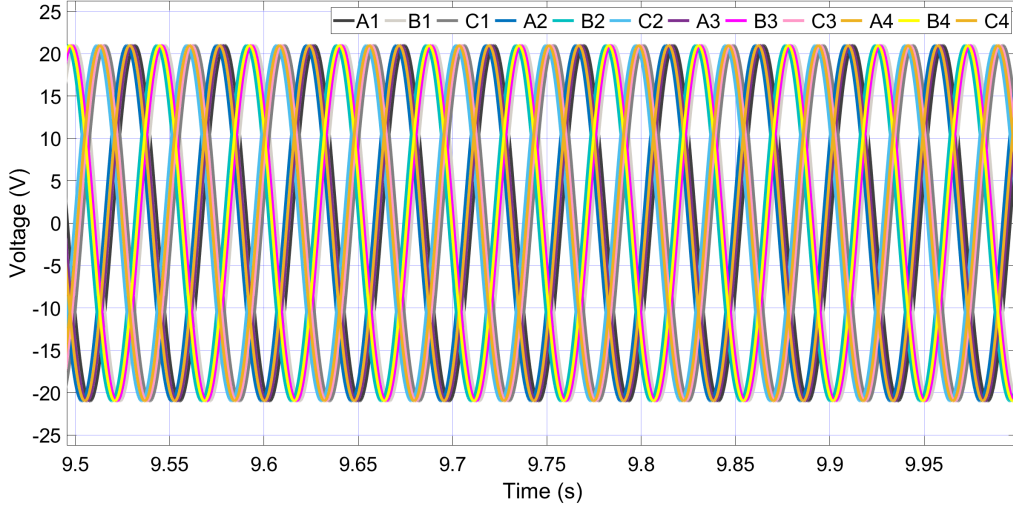


Figure 11: Consensus of the network three-phase voltages after the action of the mean-field controller.

4 Discussion

The numerical results demonstrate the effectiveness of the proposed mean-field coupling architecture in achieving frequency and phase synchronization among PMSG micro-generators with heterogeneous initial operating conditions. These findings are consistent with the theoretical framework of Benedetto et al. [1], who investigated complete phase synchronization in the mean-field limit of the Kuramoto model. While their analysis addresses the collective behavior of oscillator populations in the continuum limit, the present study examines a finite network of four micro-generators incorporating electromechanical dynamics, local state estimation, and adaptive control. The observed convergence toward a common frequency, followed by the progressive reduction of phase error, provides numerical evidence of synchronization under the operating conditions considered.

From an algorithmic perspective, the proposed formulation can be interpreted within the broader framework of Kuramoto synchronization in complex networks reviewed by Rodrigues et al. [4]. By expressing the coupling interaction through a common order parameter, the synchronization algorithm avoids the explicit evaluation of individual pairwise phase differences. For the all-to-all coupling formulation considered, this representation reduces the computational effort associated with evaluating the network interactions from $O(N^2)$ to $O(N)$. However, this reduction concerns the mathematical evaluation of the coupling terms and does not, by itself, establish equivalent improvements in communication latency or execution time on embedded hardware.

The present investigation also extends the application-oriented perspective of Rincón-Silva et al. [6], who addressed PMSG micro-generator synchronization using a low-cost embedded system based on the Kuramoto model. In contrast, the emphasis here is placed on the convergence properties of a mean-field formulation, including the relationship between phase cohesion, adaptive coupling, and closed-loop stability. The integration of SRF-dqPLL estimators enables each generator to obtain its phase and frequency information from local voltage measurements, while the shared mean-field reference coordinates the synchronization process. This distinction highlights the analytical contribution of the present work without implying that its numerical results constitute an experimental validation of the proposed architecture.

Regarding transient behavior, the adaptive-gain strategy produced a smooth convergence response without sustained oscillations in the simulated conditions. This observation is relevant to the stability concerns addressed by Ma and Shi [10], who investigated parameter-adaptive control for virtual synchronous generators under resistive-inductive grid-connected conditions. Although the two approaches involve different generator representations and control objectives, both emphasize the importance of transient stability in electrically coupled generation systems. In the present study, the gradual reduction of phase mismatch suggests favorable conditions for subsequent electrical interconnection; nevertheless, the suppression of circulating currents cannot be established without explicitly modeling and evaluating the corresponding electrical transients.

Despite these results, the analysis remains limited to a continuous-time digital twin of four PMSG units. Because the primary objective of this work is to establish the foundational continuous-time convergence of the mean-field architecture coupled with local SRF-dqPLL estimators, the present robustness evaluation focuses strictly on initial frequency mismatches. Consequently, the generator model neglects magnetic saturation, and the computation of the

mean-field centroid assumes reliable, instantaneous exchange of the estimated states among active nodes ($\tau = 0$). In practical implementations, measurement noise, communication delays, and inverter switching harmonics may affect the accuracy of the estimated phases and the resulting synchronization performance. However, a formal delay-robustness analysis lies beyond the scope of the present continuous-time study. Furthermore, although the mean-field formulation offers favorable computational scaling, its effectiveness in larger networks, under dynamic changes in network membership, and through systematic bounded-delay evaluations and hardware experiments remains to be established in future work.

5 Conclusions

This study presented analytical and numerical evaluations of a mean-field Kuramoto architecture for decentralized synchronization in PMSG micro-generation networks. The four-node digital-twin simulations demonstrated convergence from heterogeneous initial frequencies toward a common operating frequency, followed by a progressive reduction of phase error. These results support the proposed combination of local SRF-dqPLL estimation and adaptive-gain control within the phase-cohesive operating conditions considered.

The principal contribution lies in integrating a computationally efficient mean-field formulation with sensorless state estimation and adaptive synchronization control. Although the analytical (Routh-Hurwitz and Lyapunov) and numerical results establish the feasibility of the proposed approach within a phase-cohesive region under the simulated conditions, its performance in physical systems remains to be verified. Future work will therefore address embedded implementation and experimental assessment under measurement uncertainties, communication delays, and hardware-related disturbances.

Declarations

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability: The simulation models and data that support the findings of this study are available from the corresponding author upon request.

References

- [1] Dario Benedetto, Emanuele Caglioti, and Umberto Montemagno. "On the Complete Phase Synchronization for the Kuramoto Model in the Mean-Field Limit". In: *Communications in Mathematical Sciences* 13.7 (2015), pp. 1775–1786. DOI: 10.4310/CMS.2015.v13.n7.a6. URL: <https://doi.org/10.4310/CMS.2015.v13.n7.a6>.
- [2] Alysha Helmrich et al. "Centralization and decentralization for resilient infrastructure and complexity". In: *Environmental Research: Infrastructure and Sustainability* 1.2 (2021), p. 021001.
- [3] Juan A Acebrón et al. "The Kuramoto model: A simple paradigm for synchronization phenomena". In: *Reviews of modern physics* 77.1 (2005), pp. 137–185.
- [4] Francisco A Rodrigues et al. "The Kuramoto model in complex networks". In: *Physics Reports* 610 (2016), pp. 1–98.
- [5] Z. Ali et al. "Three-phase phase-locked loop synchronization algorithms for grid-connected renewable energy systems: A review". In: *Renewable and Sustainable Energy Reviews* 90 (2018), pp. 434–452.
- [6] Adan Rincón-Silva et al. "Low-Cost Embedded System for PMSG Micro-Generator Synchronization Based on the Kuramoto Model". In: *2026 12th International Conference on Control, Decision and Information Technologies (CoDIT)*. IEEE. Bari, Italy, 2026, pp. 2784–2789.
- [7] José Omar Cuzco Alao and Jonnathan Ruben Lavay Heras. "Modelado y simulación de un generador síncrono de imanes permanentes para un sistema de picogeneración hidroeléctrica". Bachelor's thesis. Universidad Politécnica Salesiana, Cuenca, Ecuador, 2022. URL: <https://dspace.ups.edu.ec/handle/123456789/22922>.
- [8] MathWorks. *Synchronous reluctance machine with sinusoidal flux distribution*. MATLAB Help Center. Accessed on February 1, 2026. 2026. URL: <https://la.mathworks.com/help/sps/ref/synchronousreluctancemachine.html>.

- [9] Giovanni Filatrella, Arne Hejde Nielsen, and Niels Falsig Pedersen. "Analysis of a power grid using a Kuramoto-like model". In: *The European Physical Journal B* 61.4 (2008), pp. 485–491. doi: 10.1140/epjb/e2008-00098-8.
- [10] Haoran Ma and Libao Shi. "Parameter-Adaptive Control Strategy for Stability Enhancement of Virtual Synchronous Generator Under Resistive-Inductive Grid-Connected Conditions". In: *Results in Engineering* (2026), p. 112687.
- [11] Katsuhiko Ogata. *Ingeniería de control moderna [Modern control engineering]*. 5ta. Madrid, España: Pearson Educación, 2010.
- [12] Dinuka Sahabandu et al. "Submodular Input Selection for Synchronization in Kuramoto Networks". In: *2020 59th IEEE Conference on Decision and Control (CDC)*. IEEE. 2020, pp. 5840–5847.
- [13] Hassan K Khalil. *Nonlinear Systems*. 3rd. Upper Saddle River, NJ, USA: Prentice Hall, 2002.