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LPV control for LPV systems: A vertex design approach

R. Galindo[1] *

[1]Faculty of Mechanical and Electrical Engineering, Autonomous University of Nuevo León, San Nicolás de los Garza, Nuevo León, México

Abstract

The closed-loop Quadratic Stability (QS) and performance of a Linear Parameter-Varying (LPV) control applied to an LPV system are investigated. The LPV control is constructed by interpolating static state feedback gains designed at the polytopic vertices of the system. The closed-loop performance of the LPV control applied to the LPV system is evaluated through the solution of Linear Matrix Inequalities (LMIs), considering both a linear-quadratic criterion and a norm-2 performance index. An estimation of the interpolation parameters is proposed, based on the distance between the actual operating point and the vertices of the polytope. Particular attention is given to the analysis of QS and performance at a "central" point of the LPV system's polytope. It is shown that, under certain given conditions, the feedback LPV system's stability and performance simplify at this point. The relationships between the stability and performance of controllers applied to the polytopic vertices of the system and the vertex-based closed-loop LPV control are highlighted. The effectiveness of the vertex approach is shown through its application to the LPV control of a buck converter, where the regulation and reference tracking control problems are considered.

Keywords— LPV Systems, LPV Control, LMI, Linear Quadratic Regulator, Vertex design

1 Introduction

The control of systems that can be modeled with multi-affine dependency of bounded time-varying parameters $\theta(t) \in \Re^q$, commonly called LPV control (see [1], [2], or [3]), are important for science and technology due to LPV systems capture more information than the Linear Time-Invariant (LTI) systems, are nearer to the non-linear system and hold many mathematical properties due to their affinity. LPV systems arise when i) the objective is to capture the system dynamics (possibly non-linear) working in different operating conditions, ii) the system has parametric uncertainty, with possibly fast time variations, iii) a non-linear system is linearized around different operating or equilibrium points, or around a bounded time-varying trajectory, and iv) a sensitivity analysis is realized. For this reason, LPV control has been applied, for instance, to coupled tanks in [4], flexible robots in [5], unmanned aerial vehicles in [6], inverted pendulums in [7], diesel machines in [8], and a long range LPV autopilot design across an entire flight envelope in [9] and [10]. Recently, [11] proposed a map to convert a nonlinear system into a global LPV system embedding the nonlinear behavior. Also, [12] presents a Linear Matrix Inequalities (LMI)-based methodology for simultaneous stabilization control and state estimation in robotic applications within the LPV setting. Convex conditions are proposed in [13] for controller synthesis of LPV Sampled-Data Systems and Quasi-LPV systems that have state-dependent parameters. In [14], a model reduction technique is presented for LPV systems by transforming a model reduction problem into an equivalent controller synthesis problem.

LPV control solves the compromise between the speed response of the controller and the control design for models that are closer to the real system. This is achieved by tuning its control strategy as a function of measured or estimated $\theta(t)$, that is, the controller $K(\theta(t))$ is a function of $\theta(t)$ (see [15]). One form to get $K(\theta(t))$ is by interpolation of LTI controllers designed at the vertices of the polytopic LPV system, guaranteeing QS and performance for the feedback LPV system based on the extensions to LPV systems (see [1], [3] and [16]), of the Theorem proposed by [17], and of the Bounded Real Lemma, respectively. These results reduce the infinite number of possible parameter values to the values of their bounds, *i.e.*, reduce the problem to a finite set of Linear Matrix Inequalities (LMI) in the polytopic vertices that are formed by the admissible trajectories of $\theta(t)$. When $\theta(t)$ changes slowly with time, the stability depends on the eigenvalues (see [1]), but it is not true when $\theta(t)$ has fast time variations.

A relation between generalized performance and Quadratic Stability (QS) can be seen in [18]. A cost function is used to frame the optimization problem at each vertex level, in a Model Predictive suboptimal Control (MPC)

*Corresponding author: rgalindoro@gmail.com, rene.galindoorz@uanl.edu.mx
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strategy for a class of Quasi-LPV systems (see [19]). In [20], preactuated multirate feedforward LPV control is used to compensate for the undershoots caused by non-minimum phase characteristics in boost converters.

In section 3, continuous LPV controllers are developed for polytopic LPV systems, designing the controllers at the vertices, which is a different approach to the standard LPV control (see [1]). These static state feedbacks guarantee stability and performance in the vertices of the LPV system, and the LPV controller is obtained by interpolation. The stability and performance of the LPV controller applied to the LPV system are guaranteed. The relation of the performance of the feedback LPV system with the performance of the closed-loop polytopic vertices is analyzed. An estimate of the interpolating parameters is proposed, based on the distance between the operating point and the vertices. In section 4, the results are applied to the LPV control of a buck converter. The LPV control is designed for the average model of the buck converter, and QS and performance are guaranteed under fast time variations of the parameters.

Notation.- $\Re$ is the set of real numbers; I_n is the identity matrix of dimensions $n \times n$, and M_i denotes $M(\theta_i)$ where θ_i is $\theta(t)$ at the $i^{th}-$vertex of the polytopic of the LPV system.

2 Problem statement

An LPV system described by state equations is,

$$\begin{cases} \dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t), \, x(t_0) \\ y(t) = C(\theta(t))x(t) + D(\theta(t))u(t) \end{cases} \tag{1}$$

where $\theta(t) \in \Re^q$ is measured or estimated continuous time function of parameters, $A(\theta(t))$, $B(\theta(t))$, $C(\theta(t))$ and $D(\theta(t))$ have a multi-affine rational dependency on the parameter $\theta(t)$, $x(t) \in \Re^n$, $u(t) \in \Re^m$ and $y(t) \in \Re^p$ are the state, the input and regulated output of the LPV system, being $\theta_{li} \leq \theta_i(t) \leq \theta_{ui}$, $i = 1, \ldots, q$, being θ_{li} and θ_{ui} finite, time-invariant and known bounds.

Remark 2.0.1. Nonlinearities are modelled as state-dependent parameters in LPV models for nonlinear non-minimum phase systems, such as aircraft, boost converters, and flexible link manipulators. Parameter-dependent Lyapunov functions ensure stability throughout the operating region, and to handle the undershoot performance of these systems, feedforward multirate techniques are used in [20], Model Predictive Control in [21], and a Lyapunov-based design that uses a current compensation can be seen in Chapter 4 of the work of [22]. Alternatively, Sliding Mode ControlÂ can be used. Their study is out of the scope of this work.

Based on the stability theory of Lyapunov, the system $\dot{x}(t) = A(\theta(t))x(t)$ is QS for the admissible variation of the parameters if $\exists Q > 0$ such that $A(\theta(t))Q + QA^T(\theta(t)) < 0$, is satisfied $\forall \theta(t)$. This result requires solving a feasibility problem subject to an infinite number of LMI. If the parametric dependency is multi-affine rational, the following Theorem reduces the problem to a finite number of LMI at the vertices of the polytopic of the LPV system (see [17]),

Theorem 2.1. *Let the family of autonomous linear systems defined by $\dot{x}(t) = A(\theta(t))x(t)$ where,*
i) the elements of $A(\theta(t))$ are multi-affine functions of the parameter vector $\theta(t)$, i.e., $a_{ij}(\theta(t))$ is affine in $\theta_j(t)$, with all the other components of $\theta(t)$ fixed;
ii) the set $\mathcal{V}$ of admissible $\theta(t)$ is a convex polyhedron,

$$\mathcal{V} := \{\theta(t) : l \leq \theta(t) \leq v, \;\; v \in \Re^q\} \tag{2}$$

where l and u are constant vectors, and the inequality is element-wise. Then, $A(\theta(t))$ is asymptotically stable $\forall \theta(t) \in \mathcal{V}$ if $\exists Q \in \Re^{n \times n}$, $Q = Q^T > 0$, such that,

$$A_i Q + Q A_i^T < 0, \;\; i = 1, \ldots, 2^q \tag{3}$$

Remark 2.1.1. Theorem 2.1 can be applied if $A(\theta(t))$ has a rational multi-affine parametric dependency, *i.e.*, $A(\theta(t)) = \frac{N(\theta(t))}{d(\theta(t))}$ where $N(\theta(t))$ is a matrix multi-affine function and $d(\theta(t))$ is a scalar multi-affine function. So, $A(\theta(t))$ is asymptotically stable $\forall \theta(t) \in \mathcal{V}$ if $\exists Q = Q^T > 0$ such that, $A_i Q + Q A_i^T < 0$ and $d(\theta_i) > 0$ for $i = 1, \ldots, 2^q$ or $A_i Q + Q A_i^T > 0$ and $d(\theta_i) < 0$ for $i = 1, \ldots, 2^q$ (see [1]). If $A(\theta(t))$ has a non-linear parametric dependency, it can be transformed to a rational multi-affine parametric dependency, introducing fictitious parameters. Using, for instance, the convex polytopic covering technique or statistical techniques (see [1]). Also, Theorem 2.1 can be applied in closed-loop if convexity and multi-affine parametric dependency are preserved. □

Two performance indices are considered, a quadratic criterion and a norm-2 guaranteed,

1. An LPV system satisfies a linear quadratic criterion (see [1]), $J=\int_0^{\infty} x^T(t)\,\Pi x(t)dt$, $\Pi>0$, if $\exists Q=Q^T>0$, such that,
$$A_iQ+QA_i^T+\Pi<0,\ \ i=1,\ldots,2^q \tag{4}$$
2. Also, from an extension to LPV systems of the Bounded Real Lemma (see [1]), the system,
$$\begin{cases} \dot{x}(t)=A(\theta(t))x(t)+B(\theta(t))\,d(t)\\ z(t)=C(\theta(t))x(t)+D(\theta(t))\,d(t)\end{cases} \tag{5}$$
satisfies, $\|\Gamma_{zd}\|_2<\gamma$, for given $\gamma>0$, where Γ_{zd} is the input-output relation from the disturbance $d(t)$ to the regulated outputs $z(t)$, if $\exists Q=Q^T>0$ such that for $i=1,\ldots,2^q$,
$$\begin{bmatrix} A_iQ+QA_i^T & B_i & QC_i^T\\ B_i^T & -\gamma^2 I & D_i^T\\ C_iQ & D_i & -I\end{bmatrix}<0 \tag{6}$$

The LPV systems described by (1) and (5) satisfy,

Assumption 1.- $x(t)$ and $\theta(t)\in\Re^q$ can be measured or estimated, and the matrices $A(\theta(t))$, $B(\theta(t))$ and $C(\theta(t))$ are polytopic functions of $\theta(t)$, that is,
$$\begin{aligned} A(\theta(t))&=\sum_{i=1}^{2^q}\alpha_i(\theta(t))A_i,\ B(\theta(t))=\sum_{i=1}^{2^q}\alpha_i(\theta(t))B_i\\ C(\theta(t))&=\sum_{i=1}^{2^q}\alpha_i(\theta(t))C_i,\ D(\theta(t))=\sum_{i=1}^{2^q}\alpha_i(\theta(t))D_i\end{aligned} \tag{7}$$
where $\alpha_i(\theta(t))>0$, $\sum_{i=1}^{2^q}\alpha_i(\theta(t))=1$, $A_1,\ldots,A_{2^q}\in\Re^{n\times n}$, $B_1,\ldots,B_{2^q}\in\Re^{n\times m}$, $C_1,\ldots,C_{2^q}\in\Re^{p\times n}$, $D_1,\ldots,$ $D_{2^q}\in\Re^{p\times m}$, and $\alpha_i(\theta(t))$ are gotten from the interpolation algorithm proposed by [23],
$$\begin{aligned} \alpha_1(\theta(t))&=\tfrac{1}{\Gamma}\textstyle\prod_{j=1}^{q}\left(\bar{\theta}_j-\theta_j(t)\right),\\ \alpha_2(\theta(t))&=\tfrac{1}{\Gamma}\textstyle\prod_{j=2}^{q}\left(\bar{\theta}_j-\theta_j(t)\right)\left(\theta_1(t)-\underline{\theta}_1\right),\\ \alpha_3(\theta(t))&=\tfrac{1}{\Gamma}\textstyle\prod_{j=3}^{q}\left(\bar{\theta}_j-\theta_j(t)\right)\left(\bar{\theta}_1-\theta_1(t)\right)\left(\theta_2(t)-\underline{\theta}_2\right),\\ &\ \ \vdots\\ \alpha_{2^q}(\theta(t))&=\tfrac{1}{\Gamma}\textstyle\prod_{j=1}^{q}\left(\theta_j(t)-\underline{\theta}_j\right)\end{aligned} \tag{8}$$
being $\Gamma:=\prod_{j=1}^{q}\left(\bar{\theta}_j-\underline{\theta}_j\right)$. Hence, $A(\theta(t))$, $B(\theta(t))$, $C(\theta(t))$, and $D(\theta(t))$ vary in the convex hull of the plant, *i.e.*, the convex envelope of a set of A_i, B_i, C_i, and D_i, $i=1,\ldots,2^q$.

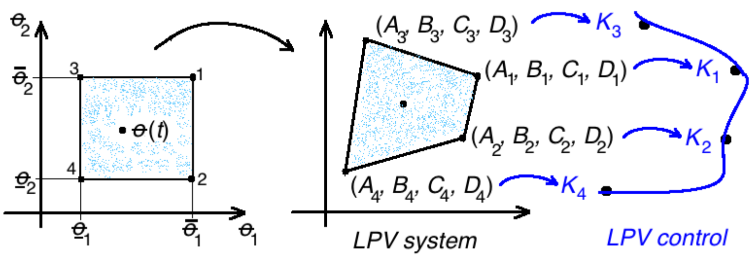

Figure 1: Convex Hull and LPV control.

Fig. 1 illustrates the convex hull of $A(\theta(t))$ with $q=2$, the vertex design of $K_1,\ \ldots,\ K_4$ controllers, and how the LPV control is obtained by interpolation.

Assumption 2.-The LPV system is subject to bounded 2−norm disturbances $d(t)\in\Re^l$, that is, $\|d(t)\|_2:=\left(\int d^T(t)d(t)dt\right)^{1/2}\leq k<\infty$, $k\in\Re$.

The problems to solve are,

Problem 1.- To guarantee QS by an LPV control applied to the LPV system given by (1), based on Theorem 2.1. The LPV controller interpolates the designed static state feedbacks at the vertices of the polytopic of the LPV system.

Problem 2.- To solve the LMI, (4) or (6), guaranteeing performance for the feedback LPV system, and solving the feasibility problem through efficient and available interior point methods.

In the following, the relation between the stability and performance of the LTI control laws at the vertices of the polytopic of the LPV system and the LPV control applied to the LPV system is investigated.

3 LPV Control

A solution to Problem 1 has been given in [24] for static state feedback and in [25] for dynamic controllers designed at each vertex. The result of [24] is further developed in,

Lemma 3.1. *Under Assumptions* 1 *and* 2, *let the LPV system described by Eq.* (1), *and suppose that the elements of* $A(\theta(t))$ *and* $B(\theta(t))$ *are multi-affine functions of the parameter vector* $\theta(t)$. *Let the LPV controller* $K(\theta(t))$ *be,*

$$K(\theta(t)) = \sum_{i=1}^{2^q} \alpha_i(\theta(t)) K_i \tag{9}$$

where $K_1,\ldots,K_{2^q}$ *are static state feedbacks at the vertices, and the state feedback,*

$$u(t) \quad = -K(\theta(t))\, x(t) + r(t) \tag{10}$$

be applied to the LPV system, where $r(t)$ *is a new reference input. Then, the closed-loop system is QS,* $\forall \theta(t) \in \mathcal{V}$, *if* $\exists Q = Q^T > 0$ *such that,*

$$A_{cli}Q + QA_{cli}^T < 0, \;\; i = 1,\ldots,2^q \tag{11}$$

where $A_{cli} = A_i - B_iK_i$ *is a Hurwitz matrix, for* $i = 1,\ldots,2^q$. *Moreover, if* $B(\theta(t))$ *is not a function of* $\theta(t)$, *and if the system operates at the "central" point,* $\alpha(\theta(t)) := \alpha_1(\theta(t)) = \cdots = \alpha_{2^q}(\theta(t))$, *then the closed-loop system is QS if,*

$$Re\left[\lambda_j\{A_s\}\right] < 0, j = 1,\ldots,n \tag{12}$$

where $A_s := \sum_{i=1}^{2^q} A_{cli}$.

Proof. Using the state feedback given by Eq. (10), the state matrix of the closed-loop LPV system $A_{cl}(\theta(t)) = A(\theta(t)) - B(\theta(t))K(\theta(t))$, at the $i^{th}-$vertex is the Hurwitz matrix A_{cli}, then applying Theorem 2.1 the feedback system is QS when inequality (11) is satisfied. Moreover, if $B(\theta(t))$ is not a function of $\theta(t)$, and if $\alpha(\theta(t)) := \alpha_1(\theta(t)) = \cdots = \alpha_{2^q}(\theta(t))$, then,

$$A_{cl}(\theta(t)) = \alpha(\theta(t)) A_s \tag{13}$$

So, since $\alpha(\theta(t)) > 0$, then applying Theorem 2.1, the closed-loop system is QS if $\exists Q = Q^T > 0$, such that inequality, $A_sQ + QA_s^T < 0$ is satisfied, that is true if A_s is a Hurwitz matrix. □

Remark 3.1.1. Inequality (11) assures QS, since if this condition is satisfied, then it holds for all linear combinations in a convex set. However, stability at the vertices of the closed-loop system is a necessary condition but not sufficient to guarantee QS, since a single Q is required for the overall system in inequality (11). Necessity follows from the superposition principle. □

If i) $B(\theta(t))$, $C(\theta(t))$, and $D(\theta(t))$ are not functions of $\theta(t)$, ii) the system operates at the "central" point, $\alpha(\theta(t)) := \alpha_1(\theta(t)) = \cdots = \alpha_{2^q}(\theta(t))$, and iii) $K_1,\ldots,K_{2^q}$ are such that $F := A_{cl1} = \cdots = A_{cl2^q}$, then from inequality (12) of Lemma 3.1 the feedback system is QS if $\exists Q = Q^T > 0$ such that $FQ + QF^T < 0$, that is true since F is a Hurwitz matrix. So, it is a case where the stability at the vertices is a necessary and sufficient condition, the feasibility problem of Theorem 2.1 does not need to be solved, and allows us to take advantage of the control designs at the vertices. This case gives insights into the LPV system stability and performance, but it is not useful for control design since it is difficult to satisfy iii), instead $K_1,\ldots,K_{2^q}$ can assign the same closed-loop dynamics at each vertex, that is, for $j = 1,\ldots,n$

$$\begin{array}{c} \lambda_j\{A_{cl1}\} = \cdots = \lambda_j\{A_{cl2^q}\} \\ \mathrm{Re}\left[\lambda_j\{A_{cl1}\}\right] < 0, \;\cdots, \mathrm{Re}\left[\lambda_j\{A_{cl2^q}\}\right] < 0 \end{array} \tag{14}$$

Remark 3.1.2. The stability condition (14) is a necessary but not sufficient condition to assure QS. To illustrate that (14) is not sufficient, let a counterexample where $A_{cl1} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ and $A_{cl2} = \begin{bmatrix} 0 & -2 \\ 0.5 & -2 \end{bmatrix}$ are Hurwitz matrices, $B(\theta(t))$ is not a function of $\theta(t)$, and the system operates at the "central" point. Hence, $A_s = \begin{bmatrix} 0 & -1 \\ -0.5 & -4 \end{bmatrix}$ is not a Hurwitz matrix and from inequality (12), QS is not assured. □

The following example illustrates a special case in which K_1 and K_2 are such that $A_{cl1} = A_{cl2}$,

Example 1.- Let the damped mass-spring system of Fig. 2 be modeled by the state-space description,

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \frac{-\theta(t)}{m} & \frac{-b}{m} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t) \tag{15}$$

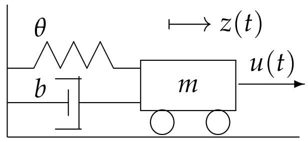

Figure 2: mass-spring system.

where m is a mass, b is a viscous friction coefficient, $\theta(t) \in [1, 3]$ is a spring coefficient, $u(t)$ is an external force applied to m, $x(t) := \begin{bmatrix} z(t) & \dot{z}(t) \end{bmatrix}^T$ is the state, being $z(t)$ the position of m. Let,

$$K_1 := \begin{bmatrix} m-1 & 2m-b \end{bmatrix} \text{ and } K_2 := \begin{bmatrix} m-3 & 2m-b \end{bmatrix} \tag{16}$$

Hence, at the $i^{th}-$vertex, the closed-loop state matrix is,

$$A_{cli} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \tag{17}$$

So, K_1 and K_2 assign the desired characteristic polynomial $s^2 + 2s + 1$ at each vertex. Hence, from Eq. (8), the interpolating parameters are,

$$\alpha_1(\theta(t)) = \tfrac{3-\theta(t)}{2}, \alpha_2(\theta(t)) = \tfrac{\theta(t)-1}{2} \tag{18}$$

and from Eq. (9), the gain of the LPV control is,

$$K(\theta(t)) = \begin{bmatrix} m-\theta(t) & 2m-b \end{bmatrix} \tag{19}$$

that in this specific example assigns the desired dynamics and the closed-loop system becomes an LTI stable system. □

In open-loop, equation (7) simplifies if $A(\theta)$ or $B(\theta)$ or $C(\theta)$ or $D(\theta)$ are not functions of $\theta(t)$, or if $\alpha_1(\theta(t)) = \cdots = \alpha_{2^q}(\theta(t))$, however, in closed-loop $K(\theta)$ given by Eq. (9) is in general a function of $\theta(t)$. This requires that $\theta(t)$ must be measured, known, or estimated. In this work, an estimation of $\theta(t)$ is proposed at the end of this Section.

Let $B(\theta)$ and $D(\theta)$ be not functions of $\theta(t)$, then the closed-loop system is,

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2^q} \alpha_i(\theta(t)) A_{cli} x(t) + Bu(t), \; x(t_0) \\ y(t) = \sum_{i=1}^{2^q} \alpha_i(\theta(t)) C_{cli} x(t) + Du(t) \end{cases} \tag{20}$$

where $A_{cli} := A_i - BK_i$ and $C_{cli} := C_i - DK_i$. The state-transition matrix depends on $\alpha_i(\theta(t))$ and the output is a linear combination of the outputs at the $i^{th}-$vertex. Also, since $\sum_{i=1}^{2^q} \alpha_i(\theta(t)) = 1$, then the system becomes time-invariant if $A_{cl1} = \cdots = A_{cl2^q}$ and $C_{cl1} = \cdots = C_{cl2^q}$, that is difficult to acomplish. Also, at the “central" point, the closed-loop system simplifies to,

$$\begin{cases} \dot{x}(t) = \alpha(\theta(t)) A_s x(t) + Bu(t), \; x(t_0) \\ y(t) = \alpha(\theta(t)) C_s x(t) + Du(t) \end{cases} \tag{21}$$

where $A_s := \sum_{i=1}^{2^q} A_{cli}$ and $C_s := \sum_{i=1}^{2^q} C_{cli}$.

Remark 3.1.3. Stability at the vertices of a closed-loop LPV system is not sufficient to guarantee QS. In the same way, “good" performance at the vertices of a closed-loop LPV system is not sufficient to guarantee “good" performance for the overall system, due to the single Q, and pre-specified Π and γ in inequalities (4) and (6). □

A solution to Problem 2 is given by,

Theorem 3.2. *Under Assumptions* 1 *and* 2, *let the LPV system be described by equations* (1) *or* (5), *and suppose that the elements of $A(\theta(t))$ and $B(\theta(t))$ are multi-affine functions of the parameter vector $\theta(t)$. Let $K(\theta(t))$ be given by Eq.* (9), *and the state feedback, $u(t) = -K(\theta(t))\,x(t) + r(t)$ be applied to the LPV system. Then, the closed-loop LPV system satisfies the linear quadratic criterion, $J = \int_0^\infty x^T(t)\,\Pi x(t)dt$, where $\Pi > 0$, if $\exists Q = Q^T > 0$ such that,*

$$A_{cli}Q + QA_{cli}^T + \Pi < 0, \;\; i = 1, \ldots, 2^q \tag{22}$$

where $A_{cli} = A_i - B_iK_i$ are Hurwitz matrices, for $i = 1,\ldots,2^q$. Also, the closed-loop LPV system satisfies $\|\Gamma_{zw}\|_2 < \gamma$ for given $\gamma > 0$, where Γ_{zw} is the input-output relation from the disturbance $w(t)$ to the regulated output $z(t)$, if $\exists Q = Q^T > 0$ such that for $i = 1,\ldots,2^q$,

$$\begin{bmatrix} A_{cli}Q + QA_{cli}^T & B_i & QC_{cli}^T \\ B_i^T & -\gamma^2 I & D_i^T \\ C_{cli}Q & D_i & -I \end{bmatrix} < 0 \tag{23}$$

where $C_{cli} = C_i - D_iK_i$. Moreover, if $B(\theta(t))$ is not a function of $\theta(t)$, and if the system operates at the "central" point, $\alpha(\theta(t)) := \alpha_1(\theta(t)) = \cdots = \alpha_{2^q}(\theta(t))$, then the closed-loop LPV system satisfies a linear quadratic criterion, if $\exists Q = Q^T > 0$ such that,

$$A_sQ + QA_s^T + 2^q\Pi < 0 \tag{24}$$

where $A_s := \sum_{i=1}^{2^q} A_{cli}$. Also, if $B(\theta(t))$ and $D(\theta(t))$ are not functions of $\theta(t)$, at the "central" point, the closed-loop LPV system satisfies $\|\Gamma_{zw}\|_2 < \gamma$ if $\exists Q = Q^T > 0$ such that,

$$\begin{bmatrix} A_sQ + QA_s^T & 2^qB & QC_s^T \\ 2^qB^T & -2^q\gamma^2 I & 2^qD^T \\ C_sQ & 2^qD & -2^qI \end{bmatrix} < 0 \tag{25}$$

where $C_s := \sum_{i=1}^{2^q} C_{cli}$.

Proof. Using the state feedback given by Eq. (10), $A_{cl}(\theta(t)) = A(\theta(t)) - B(\theta(t))K(\theta(t))$, at the $i^{th}-$vertex is the Hurwitz matrix A_{cli}. Then, from inequality (4), the feedback LPV system satisfies the linear quadratic criterion, $J = \int_0^\infty x^T(t)\Pi x(t)dt$, when inequality (22) is satisfied. Also, from inequality (6), the closed-loop LPV system satisfies $\|\Gamma_{zw}\|_2 < \gamma$, when inequality (23) is satisfied. Moreover, if $B(\theta(t))$ is not a function of $\theta(t)$, and if $\alpha(\theta(t)) := \alpha_1(\theta(t)) = \cdots = \alpha_{2^q}(\theta(t))$, then $A_{cl}(\theta(t)) = \alpha(\theta(t))A_s$ and $\sum_{i=1}^{2^q}\alpha(\theta(t)) = 2^q\alpha(t) = 1$. Hence, from inequality (4), the feedback LPV system satisfies a linear quadratic criterion if $\exists Q = Q^T > 0$, such that,

$$\alpha(\theta(t))\left(A_sQ + QA_s^T + 2^q\Pi\right) < 0 \tag{26}$$

So, since $\alpha(\theta(t)) > 0$, then the closed-loop system satisfies a linear quadratic criterion when inequality (24) is satisfied. Also, since $\sum_{i=1}^{2^q}\alpha(\theta(t)) = 2^q\alpha(t) = 1$, if $B(\theta(t))$ and $D(\theta(t))$ are not functions of $\theta(t)$, at the "central" point, from inequality (6), the closed-loop LPV system satisfies $\|\Gamma_{zw}\|_2 < \gamma$ if $\exists Q = Q^T > 0$ such that,

$$\alpha(\theta(t))\begin{bmatrix} A_sQ + QA_s^T & 2^qB & QC_s^T \\ 2^qB^T & -2^q\gamma^2 I & 2^qD^T \\ C_sQ & 2^qD & -2^qI \end{bmatrix} < 0 \tag{27}$$

Since $\alpha(\theta(t)) > 0$, the result of inequality (27) follows. □

At the "central" point, the computational effort is reduced, since only one LMI must be satisfied in Theorem 3.2. On the other hand, the price to pay for obtaining the LPV control by interpolation of the controllers at the vertices is that $\alpha(\theta(t))$ must be measured, known, or estimated. So, an estimation of the LPV control parameter $\alpha_i(\theta(t))$ is proposed in,

Proposition 3.3. *Let the state of the LPV system $x(t)$ and the states of the LTI systems at the vertices $x_i(t)$ be measured or estimated, then $\alpha_i(\theta(t))$ at the operating point can be obtained by,*

$$\alpha_i(t) = 1 - \|x_i(t) - x(t)\|_2,\ i = 1,\ldots,2^q \tag{28}$$

where $x_i(t) = x(t)$, and $\alpha_i(t) = 1$ at the $i^{th}-$vertex.

Proposition 3.3 uses the distance between the operating point and the $i^{th}-$vertex, that is, $1 - \alpha_i(t)$. Another approach is to consider the near vertex to the operating point as the dominant dynamics. Hence, the performance of the LPV system depends mainly on the performance of the LTI system at the $i^{th}-$vertex associated with,

$$\min\{1 - \alpha_i(t)\} \tag{29}$$

The results are illustrated by the following,

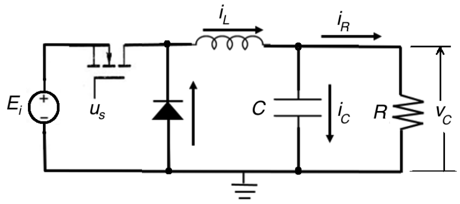

Figure 3: Buck converter.

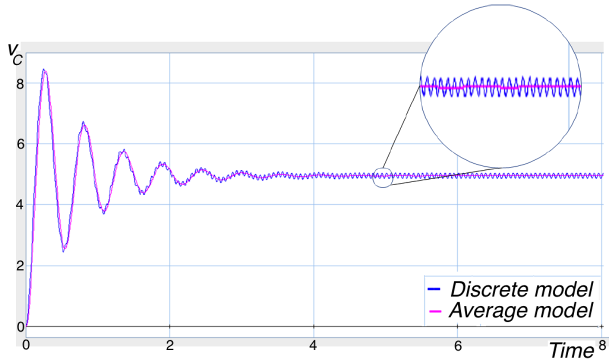

Figure 4: Voltage in the capacitor of the buck converter, discrete and average models.

4 LPV control of a buck converter

According to the work of [26], a switched model of the buck converter shown in Fig. 3 is,

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u_s(t) \tag{30}$$

where $A(\theta(t)) := \begin{bmatrix} \frac{-1}{R(t)C} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix}$, $B(\theta(t)) := \begin{bmatrix} 0 \\ \frac{E_i(t)}{L} \end{bmatrix}$, $C = 47 \times 10^{-6}$ F is the capacitance; $L = 0.01$ H is the inductance, $R(t) \in [0.1, 3]$ is the load resistance, $E_i(t) \in [0, 40]$ is the input voltage, $\theta(t) := \begin{bmatrix} E_i(t) & R(t) \end{bmatrix}^T$, $u_s(t) = 1$ and $u_s(t) = 0$ if the diode of the converter is off and on, respectively, $x(t) := \begin{bmatrix} v_c(t) & i_L(t) \end{bmatrix}^T$ is the state, that it is desired to regulate, being $v_c(t)$ the measured voltage in the capacitor C, and $i_L(t)$ the current in the inductance L. Using that fact that when $u_s(t) = 1$ and $u_s(t) = 0$, the gate-controlled switch take the values 0 and 1, respectively, and redefining $u_s(t)$ as a sufficiently smooth function taking values into the real interval [0, 1] (see [26], [27], and [28]), an LPV average model (or parametric model) of the buck converter is,

$$\dot{x}(t) = A(\theta(t))\, x(t) + B(\theta(t))\, u(t) \tag{31}$$

where $u(t)$ is the controller output, and $A(\theta(t))$ and $B(\theta(t))$ are multi-affine rational functions of $\theta(t)$, satisfying the basic assumption to get QS. Fig. 4 compares the voltage in C of the buck converter discrete and average models, where the average model captures the "slow" dynamics of the discrete model. Let $(\bar{E}_i, \bar{R})$, $(\underline{E}_i, \bar{R})$, $(\bar{E}_i, \underline{R})$, and $(\underline{E}_i, \underline{R})$ be the vertices 1, 2, 3, and 4, respectively, of the buck converter (see Fig. 1).

First, the regulation control problem and then the reference tracking control problem are considered.

4.1 Regulation

To illustrate closed-loop QS and performance under fast time variation of $\theta(t)$, a Linear Quadratic Regulator (LQR) is designed at the $i^{th}-$vertex, minimizing $J = \int \left(x^T(t)x(t) + u^T(t)u(t)\right)dt$, using the MATLAB function *lqr*. Let, $\gamma = 0.01$, and $Q \geq 0.0073029I$, hence, using the Yalmip toolbox of MATLAB, the closed-loop system is QS and has "good" performance since the feasibility problem given by inequality (23) of Theorem 3.2 has the solution,

$$Q = \begin{bmatrix} 170.79 & 2.881 \\ 2.881 & 1.144 \end{bmatrix} \tag{32}$$

Then, the LPV controller is given by Eq. (9) where $\alpha_i(\theta(t))$ is obtained by Eq. (8). This $\alpha_i(\theta(t))$ is compared with the proposed $\alpha_i(\theta(t))$ of Proposition 3.3, and is called proposed LPV control in figures 5 to 7, where the state initial

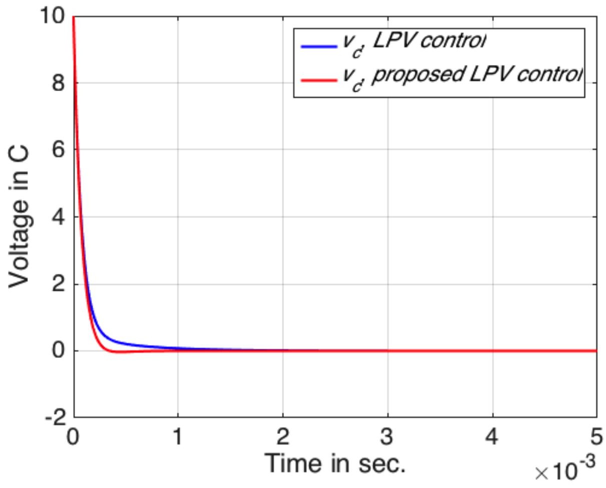

Figure 5: v_c in closed-loop.

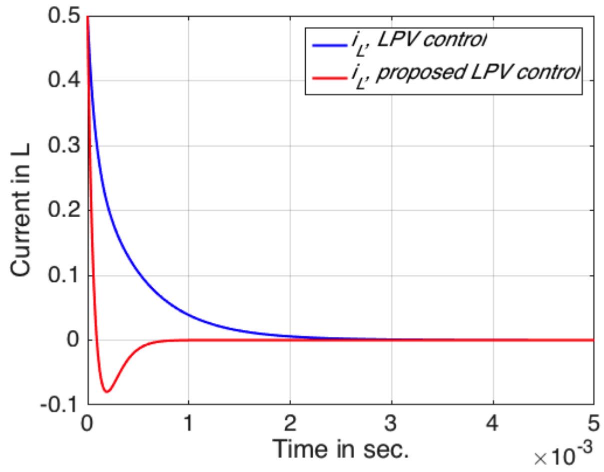

Figure 6: i_L in closed-loop.

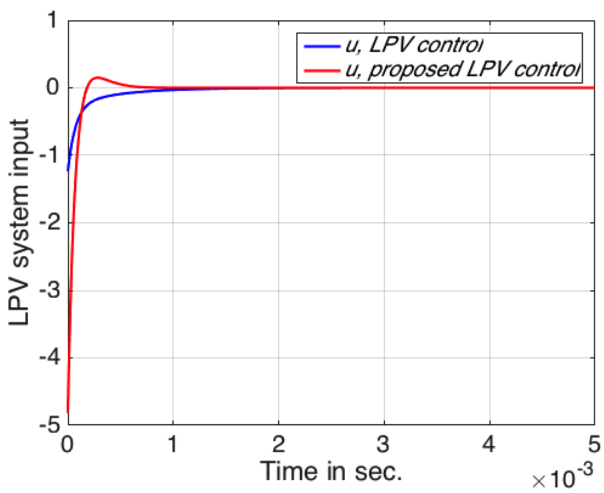

Figure 7: LPV system input u in closed-loop.

condition $x(0) := \begin{bmatrix} 10 & 0.5 \end{bmatrix}^T$, $E_i(t) = 18\sin(40\pi t)$, and $R(t) = 1.3\sin(40\pi t)$ are considered. Figures 5 to 7 show stable and "good" behavior despite the fast time variation of $E_i(t)$ and $R(t)$. The proposed LPV control applied to the buck converter has more oscillations and a faster time response than the LPV control. The advantage of the proposed LPV control is that the parameters $E_i(t)$ and $R(t)$ are not known or measured.

4.2 Reference tracking

As stated in Remark 3.1.3, “good" performance at the vertices of a closed-loop LPV system is not sufficient to guarantee “good" performance for the overall system; however, if closed-loop QS is assured, it is expected that the overall performance be similar to the one of the vertices. The above is illustrated by comparing three reference tracking control methodologies.

An LPV control called LPV control 1, considers the tracking performance index $J = \int \left(\Delta x^T(t)\Delta x(t) + u^T(t)u(t)\right)dt$ for the LQR design at the $i^{th}-$vertex. This index is minimized (see [29]), adding a dynamic system at the input reference,

$$\begin{array}{ll} \dot{\xi}_i(t) & = \left(A_i^T - P_iB_iB_i^T\right)\xi_i(t) - \bar{x}_d \\ x_d & = \sum_{i=1}^{4} \alpha_i\left(\theta(t)\right) B_i^T\xi_i(t) \end{array} \quad (33)$$

and the control law is $u(t) = -K\left(\theta(t)\right)x - x_d$, where $K\left(\theta(t)\right)$ is the LPV control designed for the regulation control, P_i is the solution of the Riccati equation solved at the $i-$ vertex in the $K\left(\theta(t)\right)$ design, and $\bar{x}_d$ is a new reference input.

An LPV control called LPV control 2 is implemented in the tracking control configuration of Fig. 8, using a translation to the origin, by the changes of variables $\Delta x := x - x_d$ and $\Delta u := u - u_d$, where $x_d := \left[\begin{array}{cc} v_d & i_d \end{array}\right]^T$ and u_d are the desired state and input, respectively, and $K\left(\theta(t)\right)$ is the LPV control designed for the regulation control. The desired input is obtained from Eq. (31) in stationary state, that is, $0 = A\left(\theta\right)x_d + B\left(\theta\right)u_d$, which implies,

$$u_d = \frac{v_d}{E_i(t)} \quad (34)$$

Proposition 3.3 is also used for LPV control 1 and is called proposed LPV control.

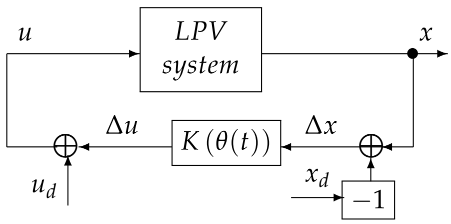

Figure 8: Tracking control configuration.

In figures 9 to 11, the state initial condition $x(0) := 0$, $E_i(t) = 15$ V, $0 \leq t \leq 0.005$, $E_i(t) = 13$ V, $0.005 \leq t$, and $R(t) = 3\ \Omega$, $0 \leq t \leq 0.005$, $R = 2\ \Omega$, $0.005 \leq t$, $x_d = \left[\begin{array}{cc} 15 & 5 \end{array}\right]^T$ for LPV control 1 and $\bar{x}_d = \left[\begin{array}{cc} 15 & 5 \end{array}\right]^T$ for LPV control 2, are considered. Closed-loop QS is assured by Theorem 3.2. Figures 9 to 11 show closed-loop stable and “good" behavior of the LPV control applied to the LPV system. These properties are similar to the closed-loop stability and performance at the vertices, that is, v_c tracks its reference until E_i changes at $t = 10$. As expected for an optimal control, in Figures 9 and 10, x has a stationary state error for $0.005 \leq t$, unless the reference changes accordingly, and in Fig. 11 the control law is smooth and with an appropriate magnitud without saturations. Notice that the reference of i_L was selected according to the reference of v_c and the maximum value of the resistive load. So, if it is desired that i_L tracks its reference, the reference of v_c must be selected accordingly. As shown in Fig. 9, LPV control 1 has bigger stationary state value for v_c than LPV control 2 or the proposed LPV control; however, as shown in Fig. 10, LPV control 1 has less stationary state value for i_L than the other two controls. The proposed LPV control has a bigger initial value for u (see Fig. 9) than the other two, with the advantage that it does not requires measurement of $\theta(t)$.

An LPV control called LPV control 3 is designed in the tracking configuration of Fig. 12, where a second loop is added with an integral action, and the optimal controls are redesigned at the vertices. Let, $z := \left[\begin{array}{cc} x_o^T & x^T \end{array}\right]^T$ and the augmented system be,

$$\begin{array}{l} \dot{z} = \left[\begin{array}{cc} 0 & -I \\ \epsilon I & A_i \end{array}\right] z + \left[\begin{array}{c} 0 \\ B_i \end{array}\right] u + \left[\begin{array}{c} I \\ 0 \end{array}\right] x_d \\ y = \left[\begin{array}{cc} 0 & I \end{array}\right] z \end{array} \quad (35)$$

where $0 < \epsilon \ll 1$ is a positive real number that was added to assure the stability of the augmented system. Then, a Linear Quadratic Regulator (LQR) is designed at the $i^{th}-$vertex, minimizing $J = \int \left(z^T(t)z(t) + u^T(t)u(t)\right)dt$, using the MATLAB function *lqr*. The state control for each vertex is $u = -\left[\begin{array}{cc} K_{oi} & K_i \end{array}\right] z$, and using the Yalmip toolbox of MATLAB, the augmented closed-loop system is QS and has “good" performance since the feasibility problem

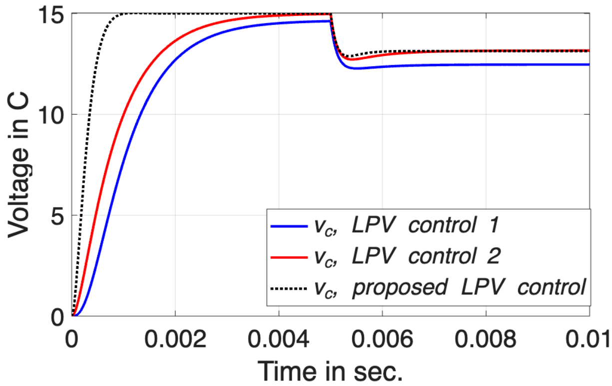

Figure 9: v_c in closed-loop.

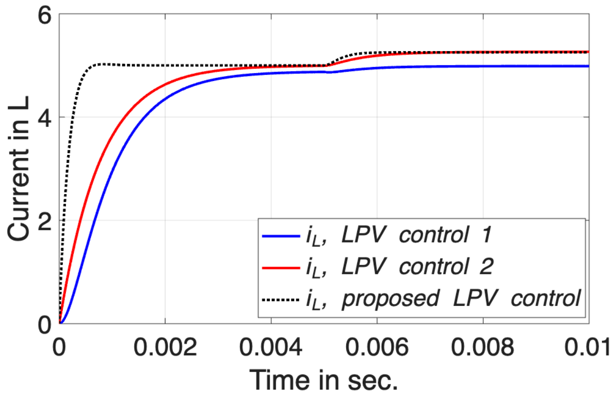

Figure 10: i_L in closed-loop.

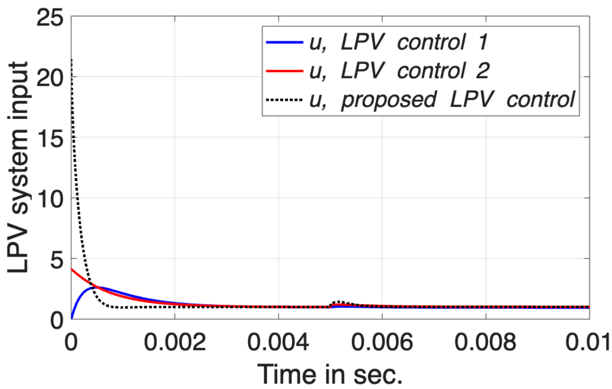

Figure 11: LPV system input u in closed-loop.

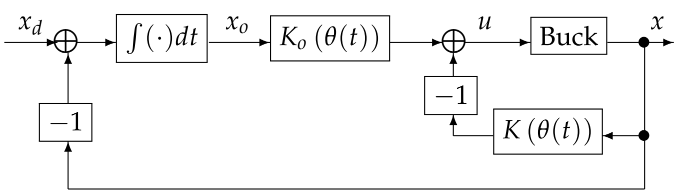

Figure 12: Tracking control configuration.

given by inequality (23) of Theorem 3.2 has a solution. Then, the LPV controller is given by Eq. (9), where $\alpha_i(\theta(t))$ is obtained by Eq. (8), that is,

$$\begin{aligned} u &= -K_o\left(\theta(t)\right)x_o - K\left(\theta(t)\right)x \\ K_o\left(\theta(t)\right) &:= \textstyle\sum_{i=1}^{2^q} \alpha_i\left(\theta(t)\right)K_{oi} \end{aligned} \tag{36}$$

where $K\left(\theta(t)\right) = \sum_{i=1}^{2^q} \alpha_i\left(\theta(t)\right)K_i$. In figures 13 to 15, the state initial condition $x(0) := 0$, $E_i(t) = 15$ V, $0 \leq t \leq 10$,

$E_i(t) = 13$ V, $10 \leq t$, and $R(t) = 3\ \Omega$, $0 \leq t \leq 10$, $R = 2.5\ \Omega$, $10 \leq t$, $x_d = \begin{bmatrix} 15 & 5 \end{bmatrix}^T$, are considered. Figures 13

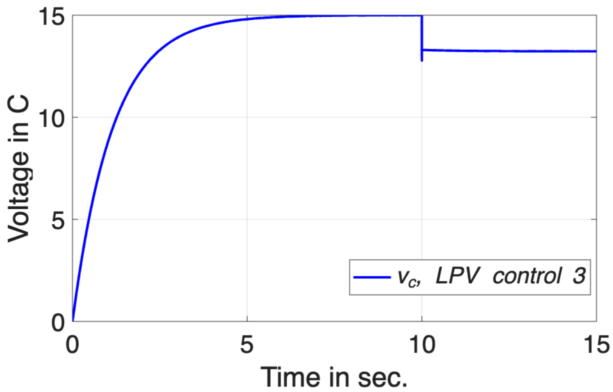

Figure 13: v_c in closed-loop.

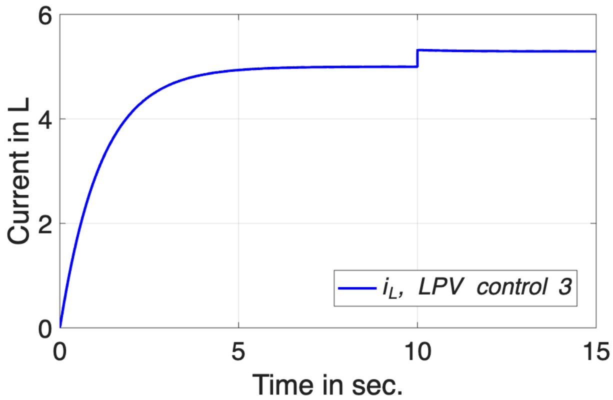

Figure 14: i_L in closed-loop.

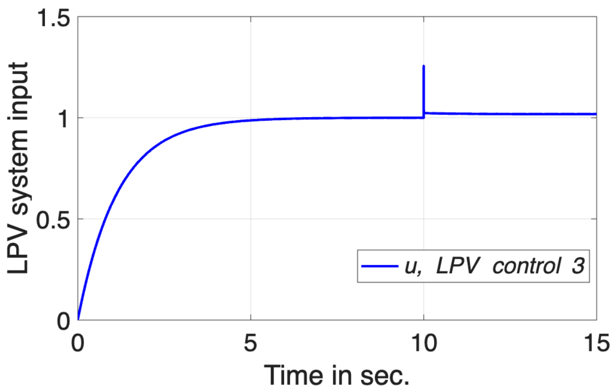

Figure 15: LPV system input u in closed-loop.

to 15 show closed-loop stable and "good" behavior of the LPV control applied to the LPV system. Fig. 13 shows that v_c tracks its reference until E_i changes at $t = 10$. In Figures 13 and 14, x has a stationary state error for $10 \leq t$, unless the reference changes accordingly, and in Fig. 15, the control law is smooth and with an appropriate magnitud without saturations. LPV control 3 has the lowest time response when compared to LPV control 1 and 2.

Finally, conclusions are given in the following section.

5 Conclusions

Solving Linear Matrix Inequalities (LMI), Quadratic Stability (QS), and performance are guaranteed for a Linear Parameter-Varying (LPV) control applied to an LPV system. The LPV control interpolates static state feedbacks designed for the polytopic vertices of the system. An LPV controller is proposed as a function of interpolating parameters estimation, which is based on the distance between the operating point and the vertices. The advantage of the proposed LPV control is that the interpolating parameters are not known or measured. In particular, QS stability and performance are assured at a "central" point of the polytopic of the LPV system. Under certain given conditions, the feedback LPV system's stability and behaviors simplify at this point. The vertex approach was successfully applied to the LPV control of a buck converter, where the state is regulated to the origin, and the output voltage tracks its reference. Other estimates of the interpolating parameters and their properties can be analyzed in future works. The results are also useful for dynamic controllers designed at the vertices. Future investigations will be about the relationships between the stability and performance of controllers applied to the vertices of the system and the closed-loop control of affine and/or nonlinear non-minimum phase systems.

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