Journal of Dynamical Systems and Complexity (2025) **1**(1), 12-19

Continuous and discrete stability criteria comparison

Baltazar Aguirre-Hernández[1]*, Edgar Cristian Díaz-González [1],
Carlos Arturo Loredo-Villalobos,[2] and Tatiana Sánchez Alcántara[1]

[1]Departamento de Matemáticas, Universidad Autónoma Metropolitana - Iztapalapa, CDMX, Mexico
[2]Departamento de Ciencias Básicas, Universidad Autónoma Metropolitana - Azcapotzalco, CDMX, Mexico

Abstract

A wide range of phenomena that evolve over time can be modeled and analyzed using both continuous and discrete-time systems. Choosing the appropriate type of system depends on the nature of the problem being studied and its constraints. For any engineer or scientist working on the design of dynamical systems, a thorough understanding of both types of systems is essential. In this paper explain some differences between the stability criteria in continuous linear systems and discrete linear systems. First, we explain which criteria have versions for both Hurwitz and Schur polynomials. Then, we present some criteria that are similar but not exactly the same version, and finally, we illustrate the differences with an important result that holds for Hurwitz polynomials but not for Schur polynomials.

Keywords— Hurwitz stability, Schur stability, Kharitonov's theorem

1 Introduction

The concept of stability in differential equations has been defined in many different ways, depending on whether the time is continuous or discrete, we have Hurwitz-type stability and Schur-type stability, respectively. One way to study the stability of a linear system is by analyzing the roots of the associated characteristic polynomial. In the case of continuous time, for the system to be stable, it is required that all roots are located in the left half of the complex plane; while for discrete time, it is required that the roots are located inside the circumference of radius one of the complex plane. There are a large number of results that allow us to determine the stability from the coefficients of the characteristic polynomial, without explicitly calculating the roots. A seminal reference that presents the study of Hurwitz stability is [1]. The criteria to be presented here are extensively analyzed in [2]. For the case of Hurwitz-type stability, the main criteria can be consulted in [3]. Other important references on details and applications of these criteria can be consulted in [4] and [5].

In this paper, we present those criteria that are applicable to both continuous time systems and discrete time systems, highlighting the slight differences in the statements of the criteria for both cases. However, there exist results that are not valid for both cases; for example, the Kharitonov theorem is valid for Hurwitz polynomials, but it is not valid for Schur polynomials. The difference of this paper compared to previous works is that in other works, such as [6] where the criteria for both cases of Hurwitz polynomials and Schur polynomials are presented, without marking the similarities and also do not mention that there are results that can occur in one case and not in the other. In the present paper, we first present criteria that have a version in both cases, that is, they have a version for both Hurwitz polynomials and Schur polynomials, such as the Phase Theorem and the Hermite-Biehler Theorem. Then, we present criteria that are not exactly the same version in both cases, but they are very similar criteria, such as the Routh-Hurwitz criterion for Hurwitz polynomials and the Schur-Cohn criterion for Schur polynomials. Finally, to illustrate the differences, we present the famous Kharitonov Theorem, which is valid for Hurwitz polynomials but not for Schur polynomials.

2 Background concepts about Hurwitz and Schur polynomials

Definition 2.1 (Hurwtiz-type stability). *A polynomial $p(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0 \in \mathbb{C}[s]$ is called Hurwitz-type, Hurwitz-stable or Hurwitz, if $s_i \in \mathbb{C}- = \{a + ib : a < 0\}$, for all root s_i of p.*

*Corresponding: bahe@xanum.uam.mx
Received: September 14, 2025, **Published:** December 17, 2025

If the roots s_i are purely imaginary, we will say that $p(s)$ is critically stable. The following simple example illustrates the previous concepts.

Example 1. The polynomial $r(s) = s^2 + (2+2i)s + (1+2i)$ has the roots: $r(s) = (s+1+2i)(s+1)$, $s = -1-2i$ and $s = -1$, therefore is a Hurwitz-stable polynomial. Whereas the polynomial $f(s) = s^2 + (-3+i)s - (-2+i)$ is neither Hurwitz nor critically stable, since its roots are: $s = 2-i$ and $s = 1$.

Definition 2.2 (Schur-type stability). *A polynomial $q(z) = a_n z^n + a_{n-1}z^{n-1} + \cdots + a_0 \in \mathbb{C}[z]$ is called Schur-type, Schur-stable or Schur, if $z_i \in \mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, for all root z_i of q.*

If the modulus of its roots, z_i, is the unit, we say that the polynomial $q(z)$ is critically stable. We will say that the polynomials $p(s)$ and $q(z)$ are unstable, if they are neither stable nor critically stable.

Example 2. The polynomial $g(z) = z^2 + (-\frac{1}{2} + \frac{1}{4}i)z - \frac{1}{8}i$ is Schur-stable because $g(z) = (z - \frac{1}{2})(z + \frac{1}{4}i)$, and its roots are: $\frac{1}{2}$ and $-\frac{1}{4}i$. Whereas the polynomial $h(z) = z^2 + (-1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)z - (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)$ is not Schur-stable since $h(z) = (z-1)(z + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)$, and its roots are $z = 1$ and $z = -$z=$-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$. However, if it is critically stable.

These definitions of stability of polynomials $p(s)$ and $q(z)$ seems to require the computation of the roots, however this is not really necessary since it is sufficient to apply some test that verifies the stability condition.

Let be $p(s)$ a complex coefficient polynomial and consider $p(i\omega)$, $\omega \in \mathbb{R}$, then the polynomial $p(i\omega) = \alpha(\omega) + i\beta(\omega)$ determines a curve on the complex plane:

$$\begin{array}{rccc} p: & \mathbb{R} & \longrightarrow & \mathbb{C} \\ & \omega & \mapsto & \alpha(\omega) + i\beta(\omega). \end{array} \tag{1}$$

Definition 2.3. *We call the argument of $p(i\omega)$, denoted by $\arg(p(i\omega))$, the phase of $p(i\omega)$.*

Definition 2.4. *Let be $q(z)$ a complex coefficients polynomial, consider $q(e^{i\theta})$, $\theta \in [0, 2\pi)$, then the polynomial $q(e^{i\theta}) = \alpha(\theta) + i\beta(\theta)$ determines a curve on the complex plane:*

$$\begin{array}{rccc} q: & [0, 2\pi) & \longrightarrow & \mathbb{C} \\ & \theta & \mapsto & \alpha(\theta) + i\beta(\theta). \end{array} \tag{2}$$

Definition 2.5. *We call the argument of $q(e^{i\theta})$, denoted by $\arg q(e^{i\theta})$, the phase of $q(e^{i\theta})$ (see [7]).*

3 Equivalent theorems for Hurwitz and Schur polynomials

3.1 The Theorem of the Phase

Theorem 3.1 (see [2]). *If $p(s) \in \mathbb{C}[s]$ is a Hurwitz polynomial with complex coefficients of degree $n \geq 1$, then $\frac{d}{d\omega} \arg p(i\omega) > 0$ for all $\omega \in \mathbb{R}$.*

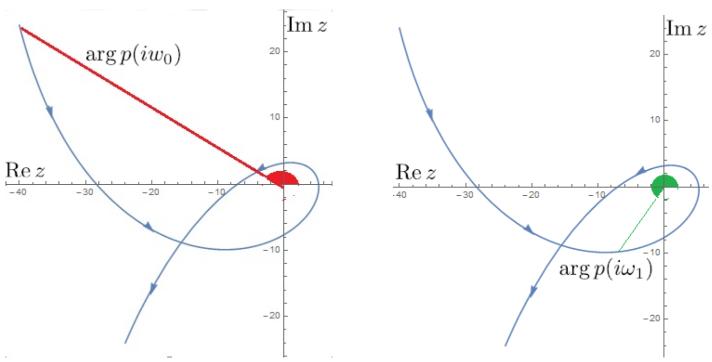

Figure 1: Phase increase illustration for polynomial $p(s) = (s+2)^2(s+1)(s+1-i)$. In blue the plot of $p(i\omega)$, red line $\arg p(i\omega_0)$ and green line $\arg p(i\omega_1)$, with $\arg p(i\omega_0) < \arg p(i\omega_1)$.

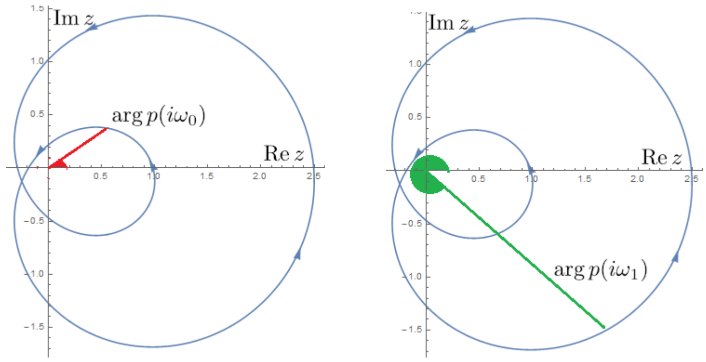

Figure 2: Phase increase illustration for polynomial $q(s) = (s + \frac{1}{2} + \frac{1}{2}i)(s + \frac{1}{4} - \frac{1}{2}i)$. In blue the plot of $p(i\omega)$, red line $\arg p(i\omega_0)$ and green line $\arg p(i\omega_1)$, with $\arg p(i\omega_0) < \arg p(i\omega_1)$.

Theorem 3.2 (see [2])**.** *If $q(z) \in \mathbb{C}[z]$ is a non-constant Schur polynomial, then $\frac{d}{d\theta} \arg q(e^{i\theta}) > 0$ for all $\theta \in [0, 2\pi)$.*

The following theorem, together with its discrete analogue, shows how the number of roots of a polynomial situated in the left half-plane, respectively on the unit circle, can be determined by modifying the argument of the polynomial along its stability boundary.

Theorem 3.3. *Let be $p(s) \in \mathbb{C}[s]$ an n degree polynomial without zeroes on the imaginary axis. Then,*

$$\Delta_{-\infty}^{\infty} \arg p(i\omega) = (n - 2v)\pi, \tag{3}$$

where $\Delta_{-\infty}^{\infty} \arg p(i\omega)$ is the net increase of $\arg p(i\omega)$ from $-\infty$ to ∞, and v is the number of zeros of $p(s)$ in the right open half-plane, taking into account its multiplicities. In particular, $p(s)$ is Hurwitz-stable if and only if

$$\Delta_{-\infty}^{\infty} \arg p(i\omega) = n\pi. \tag{4}$$

Proof. The proof of the necessary condition is based in ideas of [8] and [2], but we provide a more detailed proof so that it can be better understood. According to the fundamental theorem of algebra we have

$$p(t) = a_n(t - a_1 - ib_1)(t - a_2 - ib_2) \cdots (t - a_n ib_n), \tag{5}$$

then

$$p(i\omega) = a_n[-a_1 + i(\omega - b_1)][-a_2 + i(\omega - b_2)] \cdots [-a_n + i(\omega - b_n)]. \tag{6}$$

Next, we calculate the phase of $p(i\omega)$

$$\begin{aligned} \arg p(i\omega) &= \arg(a_n) + \arg[-a_1 + i(\omega - b_1)] + \ldots + \arg[-a_n + i(\omega - b_n)] \\ &= \arg(a_n) + \arctan\left(\frac{\omega - b_1}{-a_1}\right) + \ldots + \arctan\left(\frac{\omega - b_n}{-a_n}\right). \end{aligned} \tag{7}$$

Subsequently, we derive

$$\frac{d}{d\omega}[\arg p(i\omega)] = \frac{1}{1 + \left(\frac{\omega - b_1}{-a_1}\right)^2}\left(-\frac{1}{a_1}\right) + \ldots + \frac{1}{1 + \left(\frac{\omega - b_n}{-a_n}\right)^2}\left(-\frac{1}{a_n}\right), \tag{8}$$

for all $k = 1, \ldots, n$ it holds that $a_k < 0$, since $a_k + ib_k$ is a root of the Hurwitz polynomial $p(t)$. Therefore $\arg p(i\omega)$ is an increasing function of ω. Now we evaluate the limits

$$\lim_{\omega \to +\infty} \arg p(i\omega) = \arg(a_n) + \frac{\pi}{2} + \ldots + \frac{\pi}{2} = \arg(a_n) + \frac{n\pi}{2} \tag{9}$$

$$\lim_{\omega \to -\infty} \arg p(i\omega) = \arg(a_n) - \frac{\pi}{2} - \ldots - \frac{\pi}{2} = \arg(a_n) - \frac{n\pi}{2}. \tag{10}$$

Thus

$$\arg[p(+i\infty)] - \arg[p(-i\infty)] = n\pi. \tag{11}$$

The proof of the sufficiency can be found at [8]. □

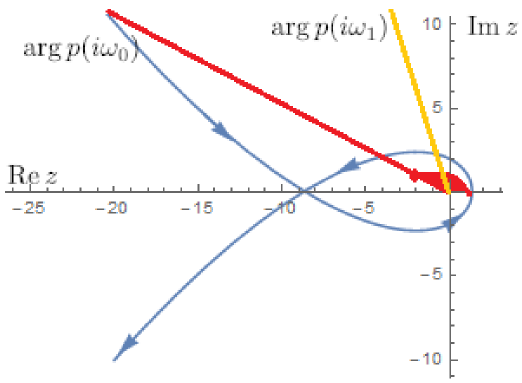

Figure 3: Argument illustration for polynomial $g(i\omega)$. The plot of $g(i\omega)$ in blue, red line is $\arg g(i\omega_0) > \pi/2$ and yellow line is $\arg g(i\omega_1) < 7\pi/2$. The argument has completed a full turn around the origin when the red line increases until it reaches the yellow line.

Example 3. Consider the polynomial

$$g(t) = t^3 + (3-i)t^2 + (3-2i)t + (1-i) \tag{12}$$

then

$$\begin{aligned} g(i\omega) &= (i\omega)^3 + (3-i)(i\omega)^2 + (3-2i)(i\omega) + (1-i) \qquad (13)\\ &= -3\omega^2 + 2\omega + 1 + i(-\omega^3 + \omega^2 + 3\omega - 1) \\ \Rightarrow \arg g(i\omega) &= \arctan\left(\frac{-\omega^3 + \omega^2 + 3\omega - 1}{-3\omega^2 + 2\omega + 1}\right) \qquad (14) \end{aligned}$$

Some observations:

- $\lim_{\omega\to-\infty} g(i\omega) = \frac{\pi}{2}$, we need to start with the branch of arctan having the image $(\frac{\pi}{2}, \frac{3\pi}{2})$.
- When the graph crosses the imaginary axis at the top, it has already passed around the origin, and the angle is $\frac{5\pi}{2}$.
- If $\lim_{\omega\to\infty} g(i\omega) = \frac{7\pi}{2}$. Therefore $\Delta_{-\infty}^{\infty} \arg g(i\omega) = \frac{7\pi}{2} - \frac{\pi}{2} = \frac{6\pi}{2} = 3\pi$, thus $g(s)$ is Hurwitz.

Theorem 3.4. *Let be $q(z) \in \mathbb{C}[z]$ an n degree polynomial, without roots on the unitary circle $\delta\mathbb{D}$, then*

$$\Delta_0^{2\pi} \arg q(e^{i\theta}) = (n - v)2\pi, \tag{15}$$

where $\Delta_0^{2\pi} \arg q(e^{i\theta})$ is the net increase of $\arg q(e^{i\theta})$ from 0 to 2π, and v is the number of roots of $q(z)$ outside $q(z)$, taking into account its multiplicities. In particular, $q(z)$ is Schur-stable if and only if

$$\Delta_0^{2\pi} \arg q(e^{i\theta}) = n2\pi. \tag{16}$$

Proof. See [2]. □

In the context of geometry, a polynomial is considered Schur-stable if and only if the frequency diagram, defined as the function $q(e^{i\theta})$ for all values of the variable $\theta \in [0, 2\pi]$, describes a curve that encircles the origin n times in a counterclockwise direction.

Example 4. Consider the polynomial

$$h(z) = z^3 + (-\frac{1}{2} + i\frac{1}{2})z^2 + (-\frac{1}{4} + i\frac{1}{2})z + (\frac{1}{8} + i\frac{1}{8}) \tag{17}$$

Analyzing the frequency plot: $\theta \to h(e^{i\theta})$, for $\theta \in [0, 2\pi]$, we have

$$\begin{aligned} h(e^{i\theta}) \quad &= \cos 3\theta - \tfrac{1}{2}\cos 2\theta - \tfrac{1}{2}\sin 2\theta - \tfrac{1}{2}\sin\theta - \tfrac{1}{4}\cos\theta + \tfrac{1}{8} \\ &+ i\left(\sin 3\theta - \tfrac{1}{2}\sin 2\theta + \tfrac{1}{2}\cos 2\theta + \tfrac{1}{2}\cos\theta - \tfrac{1}{4}\sin\theta + \tfrac{1}{8}\right). \qquad (18) \end{aligned}$$

3.2 Hermite-Biehler Theorem

A real polynomial $P(z)$ of even degree satisfies the alternating property if

- p_{2m} and p_{2m-1} have the same sign.
- All roots of $P^p(\omega)$ and $P^{im}(\omega)$ are real, distinct, and furthermore the m positive roots of $P^p(\omega)$ and the $m-1$ positive roots of $P^{im}(\omega)$ are alternating, i.e:

$$0 < \omega_{e,1} < \omega_{o,1} < \cdots < \omega_{e,m-1} < \omega_{o,m-1} < \omega_{e,m}. \tag{19}$$

A similar definition is given for $P(z)$ odd.

Theorem 3.5 (Hermite-Biehler Theorem: Hurwitz case)**.** *A real polynomial $P(z)$ is Hurwitz if and only if it satisfies the alternating property.*

Proof. The proof of previous theorem can be consulted in [9]. □

Now, consider a real polynomial of n-degree

$$P(z) = p_n z^n + p_{n-1} z^{n-1} + \cdots + p_2 z^2 + p_1 z + p_0. \tag{20}$$

We define the following polynomials for $P(z)$:

$$P_s(z) = \frac{1}{2}\left[P(z) + z^n P\left(\frac{1}{z}\right)\right]. \tag{21}$$

$$P_a(z) = \frac{1}{2}\left[P(z) - z^n P\left(\frac{1}{z}\right)\right]. \tag{22}$$

Theorem 3.6 (Hermite-Biehler Theorem: Schur case)**.** *A real polynomial $P(z)$ is Schur, if and only if $P_s(z)$ y $P_a(z)$ satisfies the following*

- *$P_s(z)$ and $P_a(z)$ are polynomials of degree n with main coefficients of the same sign.*
- *$P_s(z)$ and $P_a(z)$ have only single zeros, which are located on the unit circle.*
- *On the unit circle, the zeros of $tP_s(z)$ and $P_a(z)$ alternate.*

Proof. See [8]. □

4 Similar Criteria

In this section, we present some results relating to the study of root distribution for real and complex polynomials of continuous and discrete systems, respectively.

4.1 Routh-Hurwitz criterion

Consider the n degree real polynomial in s:

$$p(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_n, \tag{23}$$

with $a_0 \neq 0$. The matrix

$$H(p) = \begin{pmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ a_0 & a_2 & a_4 & \cdots & 0 \\ 0 & a_1 & a_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_n \end{pmatrix} \tag{24}$$

is called the Hurwitz matrix of p. We now state the Routh-Hurwitz criterion, which provides necessary and sufficient conditions for determining whether a real polynomial is Hurwitz.

Theorem 4.1 (Routh-Hurwitz criterion)**.** *Let*

$$p(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_n, \tag{25}$$

an n-fixed degree polynomial, with $a_0 > 0$. Then $p(s)$ is Hurwitz if, and only if

$$a_0\Delta_1 > 0, \Delta_2 > 0, a_0\Delta_3 > 0, \Delta_4 > 0, \ldots, \begin{cases} a_0\Delta_n > 0, & \text{if } n \text{ is odd} \\ \Delta_n > 0, & \text{if } n \text{ is even} \end{cases} \tag{26}$$

where Δ_i are the principal minors of $H(p)$, i.e.

$$\Delta_1 = \det(a_1),\ \Delta_2 = \det\begin{pmatrix} a_1 & a_3 \\ a_0 & a_2 \end{pmatrix},\ \Delta_3 = \det\begin{pmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{pmatrix}, \ldots \tag{27}$$

4.2 Schur-Cohn criterion

For complex polynomials, we have Schur-Cohn criterion as the counterpart to the real case. This criterion is expressed in terms of determinants, almost in the same way as the Routh-Hurwitz criterion.

Consider the n degree complex polynomial in z:

$$p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0. \tag{28}$$

Consider the following determinants associated with the polynomial (28):

$$\Delta_k = \begin{vmatrix} a_0 & 0 & 0 & \cdots & 0 & a_n & a_{n-1} & \cdots & a_{n-k+1} \\ a_1 & a_0 & 0 & \cdots & 0 & 0 & a_n & \cdots & a_{n-k+2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{k-1} & a_{k-2} & a_{k-3} & \cdots & 0 & 0 & 0 & \cdots & a_n \\ \bar{a}_n & 0 & 0 & \cdots & 0 & \bar{a}_0 & \bar{a}_1 & \cdots & \bar{a}_{k-1} \\ \bar{a}_{n-1} & \bar{a}_n & 0 & \cdots & 0 & 0 & \bar{a}_0 & \cdots & \bar{a}_{k-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \bar{a}_{n-k+1} & \bar{a}_{n-k+2} & \bar{a}_{n-k+3} & \cdots & \bar{a}_n & 0 & 0 & \cdots & \bar{a}_0 \end{vmatrix} \tag{29}$$

where $k = 1, 2, \ldots, n$ and $\bar{a}_i$ denotes conjugate.

Theorem 4.2 (Schur-Cohn criterion)**.** *If Δ_k in (28) is non zero, for all k, then $p(z)$ has no zeros on the unit circle and the number of roots inside the circle equals the number of sign changes in the sequence of determinants Δ_1, Δ_2, ..., Δ_n.*

5 Main results: differences in interval families

In this section we present the Kharitonov theorem, which is valid for Hurwitz polynomials and we give three counterexamples for illustrating that this theorem is not valid for Schur polynomials. We have found the counterexamples for degree equal 2 and 3. The counterexample for degree 4 is due to [8].

5.1 Kharitonov's theorem

Consider the following family of polynomials

$$p(s,q) = \sum_{i=0}^{n} [q_i^-, q_i^+] s^i \tag{30}$$

where $[q_i^-, q_i^+]$ denote the i-th component close interval. We can describe $p(s,q)$ as an *interval polynomial*. Associated with the interval polynomial $p(s,q) = \sum_{i=0}^{n} [q_i^-, q_i^+] s^i$ we define the four Kharitonov polynomials:

$$\begin{aligned} K_1(s) &= q_0^- + q_1^- s + q_2^+ s^2 + q_3^+ s^3 + q_4^- s^4 + q_5^- s^5 + q_6^+ s^6 + \cdots \\ K_2(s) &= q_0^+ + q_1^+ s + q_2^- s^2 + q_3^- s^3 + q_4^+ s^4 + q_5^+ s^5 + q_6^- s^6 + \cdots \\ K_3(s) &= q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3 + q_4^+ s^4 + q_5^- s^5 + q_6^- s^6 + \cdots \\ K_4(s) &= q_0^- + q_1^+ s + q_2^+ s^2 + q_3^- s^3 + q_4^- s^4 + q_5^+ s^5 + q_6^+ s^6 + \cdots . \end{aligned} \tag{31}$$

Theorem 5.1 (see [10]). *An interval polynomial family $\mathcal{P}$ with invariant degree is Hurwitz if and only if all four Kharitonov polynomials are Hurwitz.*

Proof. The proof can be consulted in [8] and [10]. The theorem of Kharitonov is not valid for the polynomials of Schur [8]. □

5.2 Counterexamples for the Schur stability of interval families

Example 5. Consider the following interval polynomial:

$$F(z,q_0,q_1,q_2) = \left[\frac{1}{10},\frac{1}{4}\right] + \left[\frac{3}{5},1\right] z + \left[\frac{9}{10},1\right] z^2 \tag{32}$$

that is $F(z,q_0,q_1,q_2) = q_0 + q_1 z + q_2 z^2$ with $q_0 \in [1/10, 1/4]$, $q_1 \in [3/5, 1]$ and $q_2 \in [9/10, 1]$. Here, the Kharitonov's polynomials and respective roots are:

$$\begin{array}{lll} K_1(z) = \frac{1}{10} + \frac{3}{5}z + z^2; & z_1 = -\frac{3}{10} + \frac{3}{10}i, & z_2 = -\frac{3}{10} - \frac{3}{10}i \\ K_2(z) = \frac{1}{10} + z + z^2; & z_1 = \frac{1}{10}\left(-\sqrt{15} - 5\right), & z_2 = \frac{1}{10}\left(\sqrt{15} - 5\right) \\ K_3(z) = \frac{1}{4} + \frac{3}{5}z + \frac{9}{10}z^2; & z_1 = \frac{1}{6}\left(-2 + i\sqrt{6}\right), & z_2 = \frac{1}{6}\left(-2 - i\sqrt{6}\right) \\ K_4(z) = \frac{1}{4} + z + \frac{9}{10}z^2; & z_1 = \frac{1}{18}\left(-\sqrt{10} + 10\right), & z_2 = \frac{1}{18}\left(-\sqrt{10} - 10\right) \end{array} \tag{33}$$

The four polynomials K_1, K_2, K_3 and K_4 are Schur polynomials. However,

$$F(z,1/10,1,9/10) = \frac{1}{10} + z + \frac{9}{10}z^2 = \frac{9}{10}\left(z + \frac{1}{9}\right)(z+1) \tag{34}$$

is not a Schur polynomial since one of the roots is $z = -1$.

Example 6. Consider the following interval polynomial:

$$G(z,p_0,p_1,p_2,p_3) = \left[\frac{1}{4},\frac{1}{2}\right] z + \left[\frac{3}{4},1\right] z^2 + \left[\frac{3}{4},1\right] z^3, \tag{35}$$

that is $G(z,p_0,p_1,p_2,p_3) = p_0 + p_1 z + p_2 z^2 + p_3 z^3$, with $p_0 = [0,0]$, $p_1 \in [1/4, 1/2]$, $p_2 \in [3/4, 1]$ and $p_3 \in [3/4, 1]$. The Kharitonov's polynomials and respective roots are:

$$\begin{array}{llll} K_1(z) = \frac{1}{4}z + z^2 + z^3 = z\left(\frac{1}{4} + z + z^2\right); & z_0 = 0, & z_1 = -\frac{1}{2}, & z_3 = -\frac{1}{2} \\ K_2(z) = \frac{1}{2}z + z^2 + \frac{3}{4}z^3 = z\left(\frac{1}{2} + z + \frac{3}{4}z^2\right); & z_0 = 0, & z_1 = \frac{1}{3}\left(-2 + i\sqrt{2}\right), & z_3 = \frac{1}{3}\left(-2 - i\sqrt{2}\right) \\ K_3(z) = \frac{1}{4}z + \frac{3}{4}z^2 + z^3 = z\left(\frac{1}{4} + \frac{3}{4}z + z^2\right); & z_0 = 0, & z_1 = \frac{1}{8}\left(-3 + i\sqrt{7}\right), & z_3 = \frac{1}{8}\left(-3 - i\sqrt{7}\right) \\ K_4(z) = \frac{1}{2}z + \frac{3}{4}z^2 + \frac{3}{4}z^3 = z\left(\frac{1}{2} + \frac{3}{4}z + \frac{3}{4}z^2\right); & z_0 = 0, & z_1 = \frac{1}{6}\left(-3 + i\sqrt{15}\right), & z_3 = \frac{1}{6}\left(-3 - i\sqrt{15}\right). \end{array} \tag{36}$$

The four polynomials K_1, K_2, K_3 and K_4 are Schur polynomials, but

$$G(z,0,1/4,1,3/4) = \frac{1}{4}z + z^2 + \frac{3}{4}z^3 = \frac{3}{4}z(z+1)\left(z + \frac{1}{3}\right) \tag{37}$$

is not a Schur polynomial since one of its roots is $z = -1$.

Example 7. Consider the following interval polynomial:

$$p(z,q) = z^4 + \left[-\frac{17}{8},\frac{17}{8}\right] z^3 + \frac{3}{2}z^2 - \frac{1}{3}. \tag{38}$$

Here $K_1(z) = K_3(z) = p(z,-17/8)$ and $K_2(z) = K_4(z) = p(z,17/8)$. If $q = -17/8$ the polynomial $p(z,-17/8) = z^4 - 17/8z^3 + 3/2z^2 - 1/3$ has four roots: $z_{1,2} \simeq 0.786 \pm \imath 0.596$, $z_3 \simeq 0.924$ y $z_4 \simeq -0.371$ which are inside unitary circle. Likewise, if $q = 17/8$ the polynomial $p(z,17/8) = z^4 + 17/8z^3 + 3/2z^2 - 1/3$ has the roots: $z_{1,2} \simeq 0.786 \pm \imath 0.596$, $z_3 \simeq -0.924$ y $z_4 \simeq 0.371$ within unitary circle. However, the polynomial $p(z,0)$ has the roots: $z_{1,2} \simeq \pm \imath 1.303$, $z_{3,4} \simeq \pm 0.4433$ which are not inside unitary circle, and $p(z,0) = z^4 + 3/2z^2 - 1/3$ has two roots $z_{1,2} = \pm i 1.303$ outside unitary circle.

Recent information about Hurwitz and Schur polynomials can be consulted in [11] and [12].

6 Conclusions

In this paper, we have emphasized that there are criteria that have their version for both Hurwitz and Schur polynomials, such as the Phase Theorem and the Hermite-Biehler Theorem. We have also presented results that are not the exact version for both cases but that are similar or analogous, such as the Routh-Hurwitz Criterion or the Schur-Cohn Criterion. We then illustrate that there are important differences since Kharitonov's Theorem is valid for Hurwitz polynomials but not for Schur polynomials. This motivates us to study what conditions must be met for a property to be fulfilled in both cases.

Acknowledgements

The authors like to thank to the anonymous reviewers for your valuable and constructive comments and observations.

References

[1] F. R. Gantmacher. *The theory of matrices*. Vol. 131. American Mathematical Soc., 2000.

[2] D. Hinrichsen and A.J. Pritchard. *Mathematical Systems Theory I: Modelling, State Space Analysis, Stability and Robustness*. Texts in Applied Mathematics. Springer Berlin Heidelberg, 2011.

[3] B. Aguirre-Hernández et al. "Estabilidad de sistemas por medio de polinomios Hurwitz". In: *Revista de Matemática: Teoría y Aplicaciones* 24.1 (2017), pp. 61–77. DOI: 10.15517/rmta.v24i1.27751.

[4] Z. Zahreddine. "Explicit relationships between Routh-Hurwitz and Schur-Cohn types of stability". In: *Irish Math. Soc. Bull.* 29 (1992), pp. 49–54.

[5] B. R. Barmish and C. L. DeMarco. "Criteria for Robust Stability of Systems with Structured Uncertainty: A Perspective". In: *1987 American Control Conference*. 1987, pp. 476–481. DOI: 10.23919/ACC.1987.4789367.

[6] Z. Zahreddine. "Symmetric properties of Routh–Hurwitz and Schur–Cohn stability criteria". In: *Symmetry* 14.3 (2022), p. 603.

[7] J.E. Marsden and M.J. Hoffman. *Basic Complex Analysis*. W. H. Freeman, 1999.

[8] S.P. Bhattacharyya, H. Chapellat, and L.H. Keel. *Robust Control: The Parametric Approach*. Prentice-Hall information and system sciences series. Prentice Hall PTR, 1995.

[9] S.P. Bhattacharyya, A. Datta, and L.H. Keel. *Linear control theory: structure, robustness, and optimization*. CRC press, 2018.

[10] H. Parastvand et al. "Robust placement and sizing of charging stations from a novel graph theoretic perspective". In: *IEEE Access* 8 (2020), pp. 118593–118602.

[11] G. Oaxaca-Adams, R. Villafuerte-Segura, and B. Aguirre-Hernández. "On Hurwitz stability for families of polynomials". In: *International Journal of Robust and Nonlinear Control* 34.7 (2024), pp. 4576–4594.

[12] G. Oaxaca-Adams, R. Villafuerte-Segura, and B. Aguirre-Hernández. "On Schur stability for families of polynomials". In: *Journal of the Franklin Institute* 361.4 (2024), p. 106644.