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Computational training study based on a stochastic model for the currency exchange rate prediction

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Abstract

In this work, we propose a methodology to predict the exchange rate of a given currency based on a stochastic differential equation of the Black-Scholes type, which is used to train the model for a given period and thus obtain the average return and volatility parameters. The prediction is made by solving the stochastic differential equation obtained through a stochastic extension of the fourth-order Runge-Kutta method. The method is explicitly applied to the EUR-MXN and EUR-CAD exchange rates. In addition, experiments are carried out both before and after the COVID-19 pandemic to quantify the impact of the jump effect on the currency price. The results show that jump effects in predictions can be smoothed by increasing the training time, although the possible deviation from actual values would increase. On the other hand, if a better prediction is desired, it is advisable to use prediction periods and training times that are small enough to avoid jumps or instabilities.

Keywords— Stochastic differential equations, currency exchange rate, computational training, stochastic processes

1 Introduction

The analysis and prediction of financial markets emerged as a major area of development in the 20th century, leading to the establishment of financial mathematics as a well-developed discipline. Seminal contributions to option pricing by Black and Scholes [1], and later by Merton [2], highlight the central role of stochastic calculus in this field. The origins of stochastic calculus can be traced back to Louis Bachelier [3], who first modeled stock prices as a continuous random process. However, its rigorous mathematical foundation was later formalized through the works of Itô and Wiener [4, 5]. Due to its significance, a wide range of specialized books now address the applications of stochastic calculus to financial problems [6, 7, 8, 9, 10, 11].

In this work, we are interested in the problem of currency exchange rates, for which different strategies have been developed: equilibrium flow processes in exchange rates in times of crisis [12], time-varying fractal analysis [13], fuzzy logic theory [14], binary representation [15], continuous time Markov switching [16], diffusion models incorporating asymmetry between currencies [17] or diverse empirical models proposed last century [18, 19]. Specific models have also been proposed for the particular case of the USD-MXN exchange rate [20]. However, the problem of exchange rate predictability remains unresolved due to its inherent randomness and can, at best, be considered only partially addressed [21, 22]. Many proposed models are derived from the Vasicek pricing model [23], which itself builds on the seminal work of Uhlenbeck and Ornstein on Brownian motion in physics [24]. A well-known limitation of such models is their diminishing predictive power over long horizons, a shortcoming that has motivated several noteworthy modifications [25]. Another challenge arises from the occurrence of unpredictable discontinuities, commonly referred to as jumps, triggered by events such as wars, trade conflicts, or pandemics. In particular, the COVID-19 pandemic produced a jump effect that significantly impacted nearly all currencies in the world [26].

A common feature of most currency exchange rate models is that their parameters are typically estimated using regression methods based on historical data, *i.e.*, machine learning techniques [27, 28, 29, 30, 31, 32], which constitute one of the core tools of artificial intelligence (AI). However, despite the growing importance of AI, there is mounting evidence that its energy demands are increasing rapidly, largely due to the massive data processing it requires, which

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in turn generates a significant environmental cost [33, 34]. This has motivated the search for alternatives aimed at optimizing algorithms and reducing data requirements, giving rise to a new field known as *Green AI* [35]. In this context, proposals have already been made to mitigate overfitting in machine learning applications to currency exchange, such as the use of the log-distance path loss model [36]. As noted earlier, exchange rate prediction remains an open problem with multiple dimensions.

Although the aforementioned approaches have significantly contributed to understanding exchange rate dynamics, most of them emphasize increasing model complexity or incorporating additional parameters rather than evaluating the consistency of predictions under different data-training conditions. In particular, few studies have examined how the length of the training period or the presence of abrupt discontinuities—such as those caused by global crises—affect the predictive reliability of stochastic models [37, 38]. The present work seeks to address this limitation by analyzing how varying training intervals influence prediction accuracy, thereby offering new insight into the trade-off between computational effort and model performance. In this work, our objective is not to introduce a new model, but rather to explore an alternative approach that avoids excessive computational training while still enabling reliable predictions over a finite time horizon. Specifically, we seek to address the question: is it always beneficial to incorporate increasingly large amounts of historical data, or is there a point beyond which additional data no longer improve predictive performance? An interesting attempt along these lines was presented in [39] through a non-Gaussian model; however, that study predates COVID-19 and did not account for the jump effects in exchange rates caused by the pandemic. Therefore, our objective is to examine whether such jumps significantly influence predictive outcomes.

The article is organized as follows: Section 2 introduces the stochastic differential equation used as the basis for data-driven training and prediction, along with the inferential statistical techniques used to assess significant differences between training times, prediction results, and jump effects associated with COVID-19. Section 3 presents the results, followed by a discussion of the significant differences observed between the various groups of simulations presented in section 4. Finally, the article concludes with a brief summary and an outlook on future perspectives, in section 5.

2 Model and methodology

The starting point model is known as the geometric Brownian motion (GBM), which became popular after its proposal in the Black Scholes model for stock prices, which is described by the following stochastic differential equation

$$dP(t) = \alpha P(t)dt + \beta P(t)dB(t), \tag{1}$$

this equation has the following explicit solution [40, 39]

$$P(t) = P(0) \exp\left[\left(\alpha - \frac{\beta^2}{2}\right) t + \beta B(t)\right], \tag{2}$$

where $P(t)$ is the currency exchange rate at time t, $\alpha \in R$ is the average rate of return or drift, the parameter $\beta > 0$ corresponds to the volatility and $dB(t)$ is the stochastic variable or Wiener process commonly associated with Brownian motion. In fact, it is easy to demonstrate from the solution (2) that the parameter β actually corresponds to the usual definition of financial volatility, which is usually calculated with the standard deviation of the logarithmic returns [41].

The first step involves estimating the parameters α and β using historical exchange rate data for a given currency. To estimate α, we begin with the deterministic solution of Equation (1): $P(t) = P(0)\exp(\alpha t)$, which can be written in logarithmic form and by substituting the daily values of the price P_i at time t_i with the price P_0 as the initial value

$$\ln\left(\frac{P_i}{P_0}\right) = \alpha t_i, \tag{3}$$

introducing the variable $y_i = \ln\left(\frac{P_i}{P_0}\right)$, we can write the vector equation $Y = \alpha T$.

Therefore, the value α is given by

$$\alpha = \frac{T^{\intercal} Y}{||T||^2}. \tag{4}$$

On the other hand, as already mentioned, volatility β can be calculated as the standard deviation of the logarithmic returns, explicitly

$$\beta = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}}, \tag{5}$$

where $X_i = \ln (P_i / P_{i-1})$.

Once the parameters α and β have been calculated from the selected historical data period, the next step is to perform a simulation using equation (1) to predict the future exchange rate behavior. To do this, we carry out a heuristic extension of the fourth-order Runge-Kutta method to solve stochastic differential equations that has given us good results in the calculation of motion integrals of stochastic systems [42, 43].

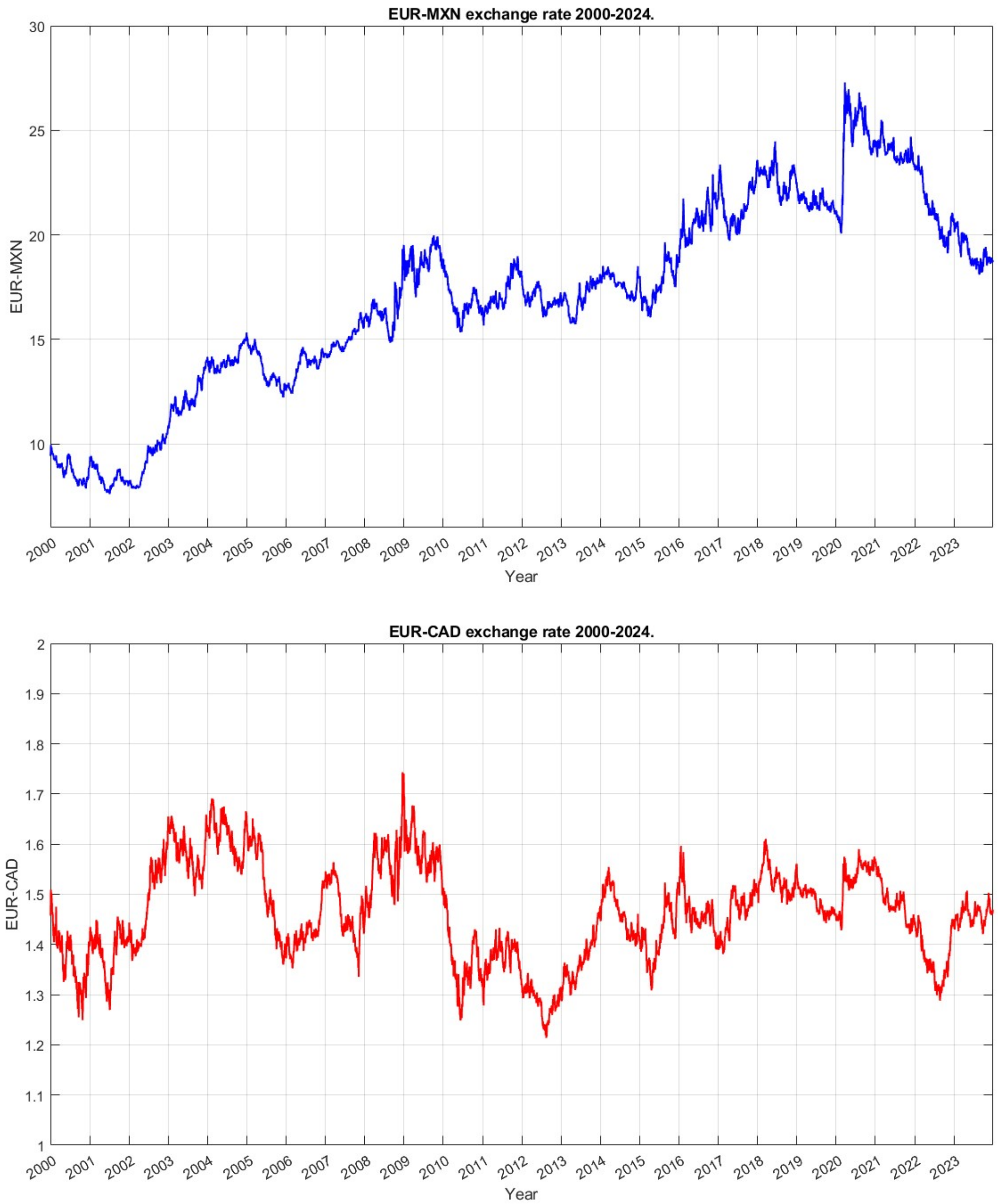


Figure 1: EUR-MXN and EUR-CAD exchange rates from 2000 to 2024.

The algorithm proposed to solve equation (1) is as follows

$$P_{n+1} = P_n + k, \tag{6}$$

where

$$\begin{aligned} k_1 &= \alpha P_n \Delta t_n + \beta P_n \Delta B_n, \\ k_2 &= \alpha(P_n + \frac{k_1}{2})\Delta t_n + \beta(P_n + \frac{k_1}{2})\Delta B_n, \\ k_3 &= \alpha(P_n + \frac{k_2}{2})\Delta t_n + \beta(P_n + \frac{k_2}{2})\Delta B_n, \\ k_4 &= \alpha(P_n + k_3)\Delta t_n + \beta(P_n + k_3)\Delta B_n, \\ k &= \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right), \end{aligned} \tag{7}$$

here Δt_n corresponds to the size of the time sub-intervals which in our case corresponds to one day, and $\Delta B_n = \sqrt{\Delta t_n} Z_n$ are the Brownian stochastic sub-intervals where Z_n is an independent random variable with normal distribution [4], in this sense to generate this random variable we use the Box-Müller algorithm [44].

It is evident that future exchange rate values cannot be known with certainty. However, past data can be used to estimate the parameters α and β, and generate predictions for recent years. For instance, data from 2010–2020 can be employed to "predict" the period 2021–2023. Given the stochastic nature of equation (1), each simulation produces different outcomes. Therefore, for every pair of parameters α and β obtained from a specific training interval, N independent simulations are performed.

For each simulation, the deviation from the actual values is calculated, after which the mean and standard deviation of these N deviations are obtained. In addition, computational training can be conducted over different historical periods to predict the same exchange rate interval. For example, models can be trained for one year, two years, and so forth, to predict the year 2023. The resulting average deviations and dispersions across these training intervals can then be compared to assess predictive performance.

In the following section, we will apply the proposed methodology to the particular cases of the EUR-MXN and EUR-CAD exchange rates, for which we will use the historical data for the period 2000-2024 shown in Figure 1 (Banco de México Foreign exchange market portal, European Central Bank Data Portal). The drastic jump caused by COVID-19 during March 2020 is evident, so the same methodology will be developed to establish pre-COVID and post-COVID predictions and analyze the differences caused by this jump.

3 Results

To apply the model and methodology described in the previous section, we carried out the study on the cases of the EUR-MXN and EUR-CAD exchange rates. Three different prediction horizons were considered: one, two, and three semesters. For each prediction period, 15 training intervals were used, ranging from one to fifteen semesters immediately preceding the prediction window. In each training–prediction configuration, we performed $N = 10{,}000$ independent runs and calculated the corresponding deviations of the predicted values with respect to the actual data. With these results, we computed the mean and standard deviation for each set of runs.

The choice of $N = 10{,}000$ simulations was based on a convergence analysis conducted during preliminary tests. We observed that both the mean and variance of the deviations reached stable values beyond approximately 8,000 runs, indicating statistical consistency in the results. Consequently, using 10,000 runs ensures robust estimates while maintaining computational efficiency, as further increasing the number of simulations produced negligible improvements in accuracy.

Additionally, as mentioned in the previous section, we divided the experiments into pre-COVID and post-COVID cases to evaluate whether the occurrence of such jumps influences prediction accuracy.

In Figure 2, we can observe the results for the pre-COVID case. Each point corresponds to the mean value of the deviation of the predicted quantities from the actual values for the $N = 10000$ runs. The results for a 6-month prediction are shown in red, for a 12-month prediction in blue, and for an 18-month prediction in green. More specifically, the period to be predicted in red was from June 30th to December 31st of 2019; in blue, it was from December 31st of 2018 to December 31st of 2019; and in green, it was from June 30th of 2018 to December 31st of 2019. In all cases, training times from 1 to 15 semesters immediately before the corresponding prediction period were used.

For example, the one-semester training period for the 6-month red forecast (June 30th to December 31st 2019) was from December 31st of 2018 to June 29th of 2019, the two-semester training period was from June 29th of 2018 to June 29th of 2019, etc. The width of the bars corresponds to the standard deviation of the N runs.

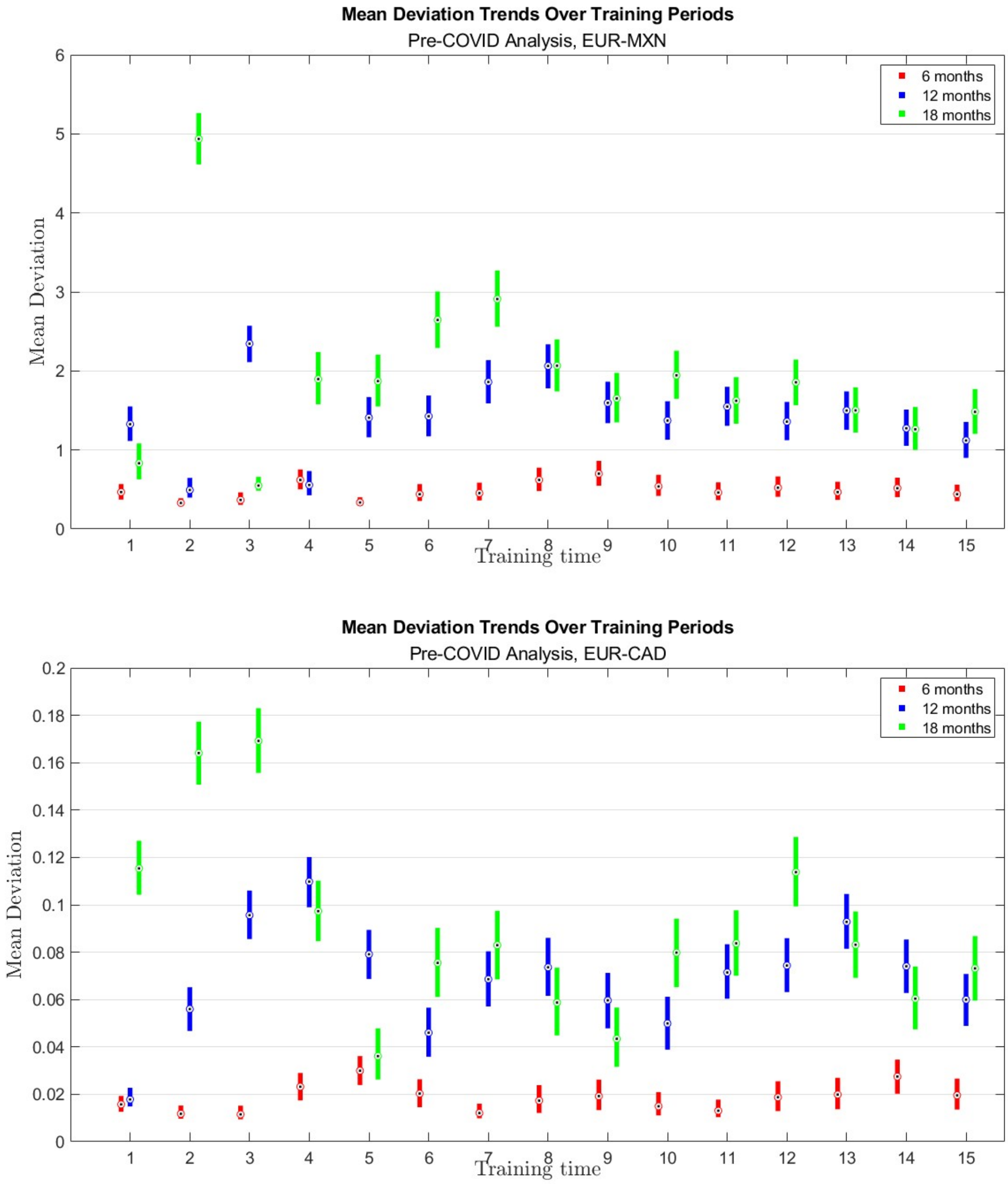
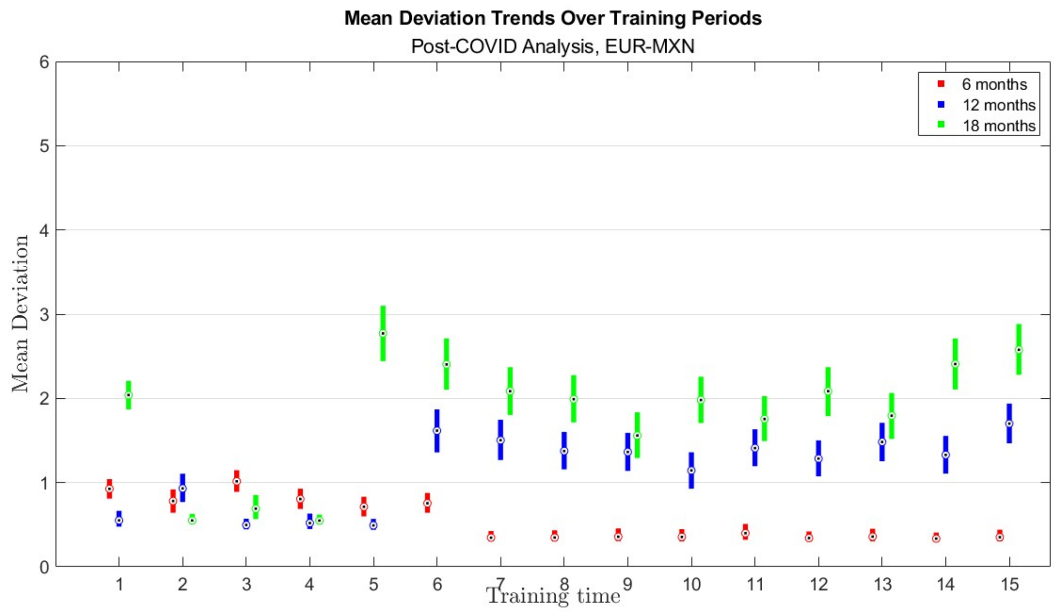


Figure 2: Pre-COVID experiment for EUR-MXN and EUR-CAD exchange rates. Where the mean deviations for 6-month predictions are shown in red, those for 12 months in blue, and those for 18 months in green. Training time units are given in semesters.

Similarly, Figure 3 shows the results of the experiment for the post-COVID case. As in the previous case, each point corresponds to the mean value of the deviations of the N runs, and the red, blue, and green points correspond to prediction periods of 6, 12, and 18 months, respectively. However, now in red the period to be predicted was from June 30th to December 31st of 2023; In blue it was from December 31st 2022 to December 31st of 2023, and in green it was from June 30th 2022 to December 31st of 2023. Training times also ranged from 1 to 15 semesters immediately before the prediction period



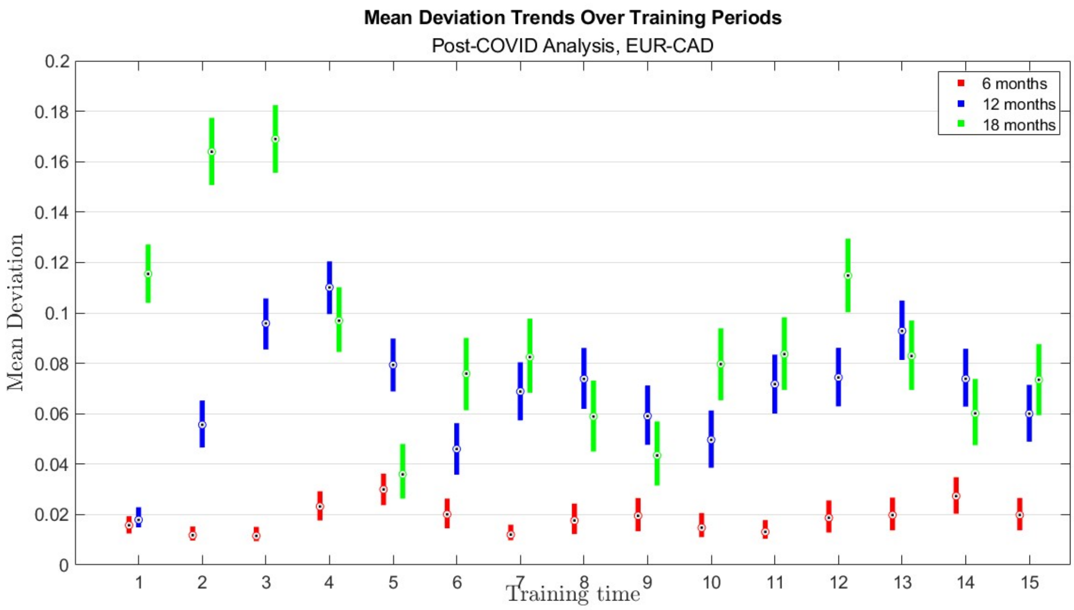


Figure 3: Post-COVID experiment for EUR-MXN and EUR-CAD exchange rates. Where the mean deviations for 6-month predictions are shown in red, those for 12 months in blue, and those for 18 months in green. Training time units are given in semesters.

4 Discussion

Let us examine in detail the results presented in Figures 2 and 3. The most striking observation across both groups is that the average deviation is consistently smaller for a prediction horizon of 6 months compared to 12 or 18 months, regardless of the exchange rate considered. Moreover, when the training period exceeds 6 semesters, the average deviations show no significant differences between the pre-COVID and post-COVID experiments. The picture changes for shorter training periods: in these cases, the deviations for 12 and 18-month predictions tend to decrease and become comparable to those for 6 months in the EUR-MXN exchange rate, particularly in the post-COVID scenario. This may be because such training windows exclude the abrupt shift in 2020, after which the EUR-MXN behaved in a relatively predictable manner, as suggested by Figure 1. By contrast, the EUR-CAD exchange rate appears more volatile during the same period. It is also noteworthy that in the pre-COVID scenario, even when excluding the 2020 spike, both exchange rates exhibited erratic behavior in the preceding years. This resulted in higher average deviations for shorter training periods and broader error bars in all cases.

Therefore, based on these experiments, the main findings can be summarized as follows:

1. When a jump occurs within the training period, it is advisable to use longer training windows to smooth out its effects. However, this comes at the cost of larger deviations in the predictions.
2. For more accurate forecasts, it is preferable to select shorter prediction horizons together with training periods of similar length. Here, "short period" refers to a sufficiently small interval in which no jumps occur and the exchange rate exhibits a relatively predictable behavior.

5 Conclusion

In this work, we proposed a training algorithm based on the Black–Scholes stochastic model to predict the price of a given currency. After estimating the parameters of average return α and volatility β over a chosen training period, we performed simulations using the solution of the corresponding stochastic differential equation, implemented through a stochastic extension of the fourth-order Runge–Kutta method. This methodology was applied to the EUR-MXN and EUR-CAD exchange rates. To assess the impact of sudden shifts, we divided the experiments into pre-COVID and post-COVID scenarios. The results indicate that when a jump occurs within the training period, extending the training window helps smooth out its effects, though at the expense of larger prediction deviations. Conversely, more accurate forecasts are obtained when both the training and prediction periods are sufficiently short, ensuring that jumps or high instability do not distort the dynamics. As a future direction, we aim to extend this methodology to the fractional case, since fractional derivatives capture memory effects, an essential feature for a more realistic description of foreign exchange markets.

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References

[1] F. Black and M. Scholes. "The pricing of options and corporate liabilities". In: *Journal of political economy* 81.3 (1973), pp. 637–654.

[2] R.C. Merton. "Theory of rational option pricing". In: *The Bell Journal of economics and management science* (1973), pp. 141–183.

[3] J. M. Courtault et al. "Louis Bachelier on the centenary of Théorie de la spéculation". In: *Mathematical Finance* 10.3 (2000), pp. 339–353.

[4] P. E Kloeden and E. Platen. *Numerical Solution of Stochastic Differential Equations*. Springer, 1992.

[5] B. Oksendal. *Stochastic differential equations: an introduction with applications*. Springer Science & Business Media, 2013.

[6] D. Lamberton and B. Lapeyre. *Introduction to stochastic calculus applied to finance*. Chapman and Hall/CRC, 2011.

[7] A. Lipton. *Mathematical methods for foreign exchange: A financial engineer's approach*. World Scientific, 2001.

[8] D. Sondermann. *Introduction to stochastic calculus for finance: a new didactic approach*. Vol. 579. Springer Science & Business Media, 2006.

[9] J.M. Steele. *Stochastic calculus and financial applications*. Vol. 1. Springer, 2001.

[10] A.N Shiryaev. *Essentials of stochastic finance: facts, models, theory*. Vol. 3. World scientific, 1999.

[11] H. Föllmer and A. Schied. *Stochastic finance: an introduction in discrete time*. Walter de Gruyter, 2011.

[12] A. Kuzmin. “Mathematical Exchange Rates Modeling: Equilibrium and Nonequilibrium Dynamics”. In: *Mathematics* 10.24 (2022), p. 4672.

[13] B. Ünal. “Time-Varying Fractal Analysis of Exchange Rates”. In: *Chaos Theory and Applications* 5.3 (2023), pp. 242–255.

[14] T. Korol. “A fuzzy logic model for forecasting exchange rates”. In: *Knowledge-Based Systems* 67 (2014), pp. 49–60.

[15] M.D. Stasiak. “Modelling of currency exchange rates using a binary representation”. In: *Information Systems Architecture and Technology: Proceedings of 37th International Conference on Information Systems Architecture and Technology–ISAT 2016–Part IV*. Springer. 2017, pp. 153–161.

[16] S. Goutte and B. Zou. “Continuous time regime-switching model applied to foreign exchange rate”. In: *Math. Finance Lett.* 2013 (2013), Article–ID.

[17] K.S. Larsen and M. Sørensen. “Diffusion models for exchange rates in a target zone”. In: *Mathematical Finance* 17.2 (2007), pp. 285–306.

[18] Y. W. Cheung, M. D. Chinn, and A. G. Pascual. “Empirical exchange rate models of the nineties: Are any fit to survive?” In: *Journal of international money and finance* 24.7 (2005), pp. 1150–1175.

[19] R.A. Meese and K. Rogoff. “Empirical exchange rate models of the seventies: Do they fit out of sample?” In: *Journal of international economics* 14.1-2 (1983), pp. 3–24.

[20] G. Bush and G. López Noria. “Uncertainty and exchange rate volatility: Evidence from Mexico”. In: *International Review of Economics & Finance* 75 (2021), pp. 704–722.

[21] L. Kilian and M.P. Taylor. “Why is it so difficult to beat the random walk forecast of exchange rates?” In: *Journal of International Economics* 60.1 (2003), pp. 85–107.

[22] B. Rossi. “Exchange rate predictability”. In: *Journal of economic literature* 51.4 (2013), pp. 1063–1119.

[23] O. Vasicek. “An equilibrium characterization of the term structure”. In: *Journal of financial economics* 5.2 (1977), pp. 177–188.

[24] G.E. Uhlenbeck and L.S. Ornstein. “On the theory of the Brownian motion”. In: *Physical review* 36.5 (1930), p. 823.

[25] T. Serafin et al. “Averaged-calibration-length prediction for currency exchange rates by a time-dependent Vasicek model”. In: *Theoretical Economics Letters* 10.3 (2020), pp. 579–599.

[26] P.K. Narayan. “Understanding exchange rate shocks during COVID-19”. In: *Finance Research Letters* 45 (2022), p. 102181.

[27] A. Goncu. “Prediction of exchange rates with machine learning”. In: *Proceedings of the International Conference on Artificial Intelligence, Information Processing and Cloud Computing*. 2019, pp. 1–5.

[28] M.S.A. Sarkar and U.A.M.E. Ali. “EUR/USD Exchange Rate Prediction Using Machine Learning”. In: *Int. J. Math. Sci. Comput* 8.1 (2022), pp. 44–48.

[29] K. Theofilatos, S. Likothanassis, and A. Karathanasopoulos. “Modeling and trading the EUR/USD exchange rate using machine learning techniques”. In: *Engineering, Technology & Applied Science Research* 2.5 (2012), pp. 269–272.

[30] V. Plakandaras, T. Papadimitriou, and P. Gogas. “Forecasting daily and monthly exchange rates with machine learning techniques”. In: *Journal of Forecasting* 34.7 (2015), pp. 560–573.

[31] C. Amat, T. Michalski, and G. Stoltz. “Fundamentals and exchange rate forecastability with simple machine learning methods”. In: *Journal of International Money and Finance* 88 (2018), pp. 1–24.

[32] J.F. Pfahler. “Exchange rate forecasting with advanced machine learning methods”. In: *Journal of Risk and Financial Management* 15.1 (2021), p. 2.

[33] A.L. Ligozat et al. "Unraveling the hidden environmental impacts of AI solutions for environment life cycle assessment of AI solutions". In: *Sustainability* 14.9 (2022), p. 5172.

[34] B. Brevini. "Black boxes, not green: Mythologizing artificial intelligence and omitting the environment". In: *Big Data & Society* 7.2 (2020), p. 2053951720935141.

[35] R. Verdecchia, J. Sallou, and L. Cruz. "A systematic review of Green AI". In: *Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery* 13.4 (2023), e1507.

[36] Y.L. Peng and W.P. Lee. "Data selection to avoid overfitting for foreign exchange intraday trading with machine learning". In: *Applied Soft Computing* 108 (2021), p. 107461.

[37] M. Kahssay and S. Miah. "A comparative analysis of stochastic models for stock price forecasting: The influence of historical data duration and volatility regimes". In: *Quantitative Finance and Economics* 9.3 (2025), pp. 602–630.

[38] B. Rossi. "Forecasting in the Presence of Instabilities: How Do We Know Whether Models Predict Well and How to Improve Them". In: *ERN: Forecasting & Simulation (Prices) (Topic)* (2019).

[39] G. Sikora et al. "Stochastic modeling of currency exchange rates with novel validation techniques". In: *Physica A: Statistical Mechanics and its Applications* 523 (2019), pp. 1202–1215.

[40] S.M. Ross. *Introduction to probability models*. Academic press, 2014.

[41] T.G. Andersen et al. "The distribution of realized stock return volatility". In: *Journal of financial economics* 61.1 (2001), pp. 43–76.

[42] E. Urenda-Cázares et al. "The noisy Pais–Uhlenbeck oscillator". In: *Journal of Mathematical Chemistry* 57 (2019), pp. 1314–1329.

[43] E. Urenda-Cázares, A. Gallegos, and R. Jaimes-Reátegui. "Effects of multiplicative noise on the Duffing oscillator with variable coefficients and its integral of motion". In: *International Journal of Modern Physics C* 31.07 (2020), p. 2050095.

[44] G.E. Box and M.E. Muller. "A note on the generation of random normal deviates". In: *The annals of mathematical statistics* 29.2 (1958), pp. 610–611.